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Multispecies and Stochastic issues

Comparative Evaluation of the Fisheries Policies in Denmark, Iceland and Norway

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Nordic cooperation is one of the world's most extensive forms of regional collaboration, involving Denmark, Finland, Iceland, Norway, Sweden, and three autonomous areas: the Faroe Islands, Greenland, and Åland.

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Content

Summary

The need for active public fisheries management is well established. In practice, fisheries management plans consist of a variety of different instruments. Central in these plans is, however, the harvesting strategy, i.e. how much of the resource is it optimal to catch during the period. A strategy is considered optimal if the rent (net benefit) from the fishery is maximized over the considered planning period.

To put some light on this issue, fisheries models have to be developed which include both a biological and economic part.

The aim of the project has been twofold: 1) to quantify the stochastic process producing this uncertainty for certain important fish stocks and 2) to further develop a method for determining optimal harvest quotas within the framework of a multi-species model, and, by this, implement the model in practice for the purpose of performing a comparative study of the fisheries in three Nordic countries: Denmark, Iceland and Norway. The harvesting (total allowable catch) policies for the cod and capelin/herring fisheries in these countries are compared. Indicators for stock overexploitation and harvest overexploitation are developed.

The basis for the model is the existence of a feedback model developed by Sandal and Steinshamn at NHH/SNF in Bergen. This model has both a deterministic and stochastic version, and it is the stochastic version that is given attention in this project. This model is unique in the sense that it is a feedback model with non-linear input functions. By a feedback model is meant that the optimal control (harvest) is a direct function of the state variable (stock) and is not found by forecasting. Further, a method for quantifying stochastic processes has been used for the practical implementation of the model.

It is this lack of implementation of the stochastic and the multi-species model to North-Atlantic fisheries that is the main motivation for this report. Uncertainty is obviously a key aspect of many of the North-Atlantic stocks both with respect to stock estimates and to the stock dynamics itself. We intend to concentrate on the economically most important ones, namely herring and cod in Denmark and capelin and cod in Iceland and Norway. The reason why we have chosen capelin instead of herring is that the multi-species interaction is much stronger between these two species. Danish cod and herring can be found in the North Sea. Norwegian cod is the so-called Arcto-Norwegian cod in the Barents Sea whereas Icelandic cod can be found in the ocean around Iceland. The Icelandic capelin is the stock off the coast of Iceland whereas the Norwegian capelin is the stock in the Barents Sea that is shared with Russia.

The term “feedback policy” refers to more or less complex rules to determine optimal harvest quotas given the present level of the fish stocks. The commonly used alternative to this approach is to find optimal time paths for harvest quotas; that is, to find optimal harvest as a function of time instead of as function of the observed stocks. Such open loop policies (i.e. time paths) are of very little use when we are faced with model uncertainties and other stochastic components. The proper way of dealing with economic and biological dynamic uncertainties is through some sort of feedback scheme policies. Feedback models take the prevailing fish stocks, whatever they may be, as inputs. Therefore, these models automatically respond to unexpected changes in the stocks. In this way they adapt to new situations as they unfold.

One of the main outcomes of the project has been the establishment of a stochastic feedback model where more appropriate indices of performance for comparing harvesting policies in the Nordic countries Denmark, Iceland and Norway is generated.

Another important task will be the development towards a proper model incorporating multi-species considerations. It has been increasingly recognized that biological interactions between species plays an important role in optimal fisheries management. To include such interactions in a feedback model is a complex undertaking. This aspect does not only affect the comparison between the efficiency of different fisheries policies, but it also contributes to our knowledge about how these fish stocks ought to be managed in the future.

A commonly proposed fishery management objective, which we adopt here, is to maximise the flow of expected discounted net revenue from the fishery over time, subject to the constraint implied by fish stock dynamics. Net revenue is the total revenue from fish harvesting minus the operating costs. Operating costs are a decreasing function of fish biomass and are commonly believed to be an increasing function of harvest.

In the project we have kept the quantities involved on a high level of aggregation. We have tried to keep the level of description as rough as possible keeping in mind that our objective is to provide a reliable tool for sustainable utilization of marine resources in the presence of a volatile environment both in the ecological, physical and economic sense.

The result of the project is that although there are clear signs of both harvest and stock overexploitation in all three countries, there were also significant differences. Thus, overexploitation of cod was found to be the least in Denmark but higher in Iceland and Norway. With respect to the herring fishery, however, it was the other way around and Denmark performed worst. A single-species stochastic model with a stochastic term was also applied, but the effect of stochasticity was small in this kind of model. The conclusion was therefore that more advanced stochastic modelling would be required.

The conclusions from the two-species models are somewhat opposite from what was found in the single-species case. The results from the single-species approach - which is an update of earlier work – show that the cod fishery in Iceland and Denmark should be closed and in Norway the harvest should be reduced by 2/3. For capelin/herring, the results are not biased. In the Danish case the harvest of herring could be increased somewhat. For capelin in Norway the actual harvest fluctuates around the optimal harvest level with tendency towards over harvesting, while for Iceland the actual harvest level is more or less in accordance with the optimal harvest level. The stock levels, on the other hand, are far below optimal.

Adding stochasticity to the single species model does not change the results qualitatively. This can be explained by the way uncertainty is handled technical in the model. Current development on uncertainty in fisheries management models shows that uncertainty may arise in different ways and therefore need to be handled more fundamentally. This is an area for future research.

Allowing species interaction between cod and capelin/herring provides on the other hand new results and insight. In the Danish case the two species model implies a less conservative harvesting pattern for both species. In fact, the current harvest of herring could according to the result be doubled. This is not an obvious result as the harvesting pattern in the two species model depends on competitive relationship between the species which are endogenously determined in the model. However, there is a need to explore the biological interaction between cod and herring in more detail. In the case of Iceland the predator-prey model implies more conservative harvesting pattern for both species, particularly the harvest of capelin should - compared to the single-species model and the actual harvest level – be reduced. Both for Denmark and Iceland the difference is significant and uniform over time. In the case of Norway, the predator-prey model implies a more complicated harvesting pattern, and the difference between the single-species and two-species model is not that significant. Furthermore, it is not uniform over time either. On average, however, the two-species model implies a more conservative pattern.

1. Introduction

The need for an active public fisheries management is well established (Warming 1911 and Gordon 1954). In practice, fisheries management plans consist of a variety of different instruments. Central in these plans is, however, the harvesting strategy, i.e. how much of the resource is it optimal to catch during the period. A strategy is considered optimal if the rent (net benefit) from the fishery is maximized over the considered planning period.

To put some light on this issue, fisheries models have to be developed which include both a biological and economic part.

The aim of the project has been twofold: 1) to quantify the stochastic process producing this uncertainty for certain important fish stocks and 2) to further develop a method for determining optimal harvest quotas within the framework of a multi-species model, and, by this, implement the model in practice for the purpose of performing a comparative study of the fisheries in three Nordic countries. The harvesting (total allowable catch) policies for the cod and capelin/herring fisheries in Iceland, Norway and Denmark are compared. Indicators for stock overexploitation and harvest overexploitation are developed.

In the bioeconomic literature stochastic models are much less frequent than deterministic models. Some examples of bioeconomic models with explicit stochastic processes and stochastic optimisation are Conrad (1992), Milliman et al. (1992), Kaitala (1993), Senina et al (1999) and Watson and Sumner (1999).

The basis for the models is the existence of a feedback model developed by Sandal and Steinshamn (1997a, 1997b, 2001a). This model has both a deterministic and stochastic version, and it is the stochastic version that will be given attention in this project. This model is unique in the sense that it is a feedback model with non-linear input functions. By a feedback model is meant that the optimal control (harvest) is a direct function of the state variable (stock) and is not found by forecasting. Further, a method for quantifying stochastic processes has been developed by McDonald and Sandal (1999) and this approach will be used for the practical implementation of the model.

The theoretical outline of the deterministic model has been described in Sandal and Steinshamn (1997a and 2001a). Results from practical implementation of the deterministic model have been reported in e.g. Arnason et al. (2000). It is this lack of implementation of the model to North-Atlantic fisheries, among other things, that is the main motivation for this report. Uncertainty is obviously a key aspect of many of the North-Atlantic stocks both with respect to stock estimates and to the stock dynamics itself (Ulltang, 1996; Nandram et al., 1997; Charles, 1998; Myers and Mertz, 1998; Sandberg et al., 1998; Rose et al. 2000). We intend to concentrate on the economically most important ones, namely herring and cod in Denmark, like in the previous project, and capelin and cod in Iceland and Norway. The reason why we have chosen capelin instead of herring is that the multi-species interaction is much stronger between these two species. Danish cod and herring can be found in the North Sea. Norwegian cod is the so-called Arcto-Norwegian cod in the Barents Sea whereas Icelandic cod can be found in the ocean around Iceland. The Icelandic capelin is the stock off the coast of Iceland whereas the Norwegian capelin is the stock in the Barents Sea that is shared with Russia.

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One of the main outcomes of the project has been the establishment of a stochastic feedback model where more appropriate indices of performance for comparing harvesting policies in the Nordic countries Denmark, Iceland and Norway is generated.

Another important task will be the development towards a proper model incorporating multi-species considerations. It has been increasingly recognized that biological interactions between species plays an important role in optimal fisheries management. To include such interactions in a feedback model is a complex undertaking, but we know that it is numerically tractable. Completing this task will not only affect the comparison between the efficiency of different fisheries policies, but it will also contribute to our knowledge about how these fish stocks ought to be managed in the future.

A commonly proposed fishery management objective, which we adopt here, is to maximise the flow of expected discounted net revenue from the fishery over time, subject to the constraint implied by fish stock dynamics. Net revenue is the total revenue from fish harvesting minus the operating costs. Operating costs are a decreasing function of fish biomass and are commonly believed to be an increasing function of harvest.

In the project we have kept the quantities involved on a high level of aggregation. We have tried to keep the level of description as rough as possible keeping in mind that our objective is to provide a reliable tool for sustainable utilization of marine resources in the presence of a volatile environment both in the ecological, physical and economic sense.

The result of the project is that although there are clear signs of both harvest and stock overexploitation in all three countries, there were also significant differences. Thus, overexploitation of cod was found to be the least in Denmark but higher in Iceland and Norway. With respect to the herring fishery, however, it was the other way around and Denmark performed worst. A single-species stochastic model with a stochastic term was also applied, but the effect of stochasticity was small in this kind of model. The conclusion was therefore that more advanced stochastic modelling would be required.

The conclusions from the two-species models instead of single-species models are somewhat opposite from what had been found in the single-species case. There were, in fact, signs of under-exploitation of herring in Denmark when a competition model for cod and herring was applied.

2. The Single Species and Deterministic Feedback Model: An Update

The purpose of this section is to update the results in Arnason et. al. (2000) where the cod and herring policies of Denmark, Iceland and Norway is evaluated using the basic deterministic single-species model Sandal and Steinshamn (1997a).

In order to calculate the optimal feedback rule for each country it is necessary to estimate the corresponding biological growth and economic profit functions.

The objective is to discover the time path of harvest that maximises the following functional:

$$\int_0^\infty e^{-\delta t}\Pi(h,x)dt$$

(1)

Subject to

$$\dot{x} = f(x,h),\ x(0) = x_0,\ \lim_{t\to\infty} x(t) = x^*$$

Where x represents the fish stock biomass, h the flow of harvest, Π net revenues and $f(.,.)$ is a function representing biomass growth. Dots on tops of variables are used to denote time derivatives, and δ is the discount rate. x_0 represents the initial biomass and x^* some positive (equilibrium) biomass level to which the optimal program is supposed to converge.[1]

In appendix 4 is the theoretical model is develop in more detail. The basic functions to estimate are the biomass growth functions and the profit functions.

[1] Indeed, the last constraint in (1), which can be derived as a transversality condition, may be regarded as the requirement of fishery sustainability.

2.1 Cod Fisheries

Biological growth functions

The basic function to estimate is the aggregate growth function g(x). It is assumed that the instantaneous change in stock biomass equals natural growth less harvest:

$$\frac{dx}{dt} \equiv f(x,h) = g(x) - h$$

It is not possible to estimate $g(x)$ directly, because the available data is in discrete time. Consequently, we employ the approximation:

$$g(x) = x_{t+1} - x_t + h,$$

Where the subscript t refers to years, x_t refers to biomass at the beginning of each year and h_t the harvest during the period $[t, t+1]$.

Different forms based on the logistic function were tried and in table 2.1 the results of the estimations are shown.

Table 2.1 Parameter values and statistical properties of the biological growth functions. Cod. Growth is measured in 1000 tons

	Function	Parameters	t-statistic	
Denmark (n = 40)	$rx\left(1-\frac{x}{K}\right)$	r = 0.603	4.53	**R^2 = 0.12**
		K = 1,433	-2.42[1]	F = 5.20
Iceland (n = 26)	$rx\left(1-\frac{x}{K}\right)$	r = 0.6699	8.55	**R^2=0.26**
		K = 1,988	-2.93	**F = 8.6**
Norway (n = 26)	$rx^2\left(1-\frac{x}{K}\right)$	r = 0.000665	12.64	**R^2 = 0.54**
		K = 2,473	25.28	F = 30.83

Note: r is the intrinsic growth rate and K is the carrying capacity of the stock[1]
The t-statistics refers to the parameter b in the estimated equation $g = aX+bX^2$

Economic profit functions

The generic profit function employed in the empirical model is:

$\pi(h, x) = p(h)h - C(x, h)$.

Where p(h) represents the (inverse) demand function for landed cod, and c(h,x) is the cost function associated with the harvest process. In the profit function the two functions are estimated separately.

Several forms for the demand functions were estimated for the three countries. The form adopted was:

$P(h) = a - bh$

Where h represents landings of cod and a and b are coefficients.

The results of the estimations are shown in table 2.2.

Table 2.2 Parameter values and statistical properties of the demand functions. Cod. Prices are measured in NOK/kg

	Function	Parameters	t-statistic	
Denmark	$p(h) = a - bh$	*a* = *18.66*	15.19	R^2 = 0.7385
(n=23)		*b* = 0.006344	-2.57	F = 53.644
Iceland	$p(h) = a - b$	*a* = *20.96*	5.46	R^2 = 0.096
(*n*=24)		*b* = 0.0426	-2.45	F = 6.02
Norway	$p(h) = a - b$	*a* = *12.65*	9.7	R^2 = 0.59
(n = 11)		*b* = 0.00839	3.94	F = 15.6

For the harvesting cost function the following functional form was adopted for all three countries:

$$C(h,x) = \alpha \frac{h^{\beta}}{x}$$

Where α and β are parameters. The dependent variable, i.e. costs, is defined as total costs less depreciation and interest payments. This may be regarded as an approximation to total variable costs. The two step procedure is applied. First the parameter β is found, where the likelihood is highest. This parameter is then exogenous given in the second step where α is estimated. The results are shown in Table 2.3.

Table 2.3 Parameter values and statistical properties of the cost functions. Cod. Costs are measured in million NOK.

	Function	parameters	t-statistic	
Denmark (n=10)	$C(h,x) = \alpha \frac{h^{1.069}}{x}$	*α = 3886.426*	16.32	R^2 = 0.7952
Iceland (*n*=152)	$C(h,x) = \alpha \frac{h^{1.1}}{x}$	*α = 5363.179*	6.45	R^2 = 0.43
Norway (n = 8)	$C(h,x) = \alpha \frac{h^{1.1}}{x}$	*α = 5848.1*	44.7	R^2 = 0.95

2.2 Capelin and Herring

The three functions for Capelin and Herring are shown in Tables 2.4, 2.5 and 2.6.

Table 2.4 Parameter values and statistical properties of the biological growth functions. Capelin/Herring. Growth is measured in 1000 tons.

	Function	parameters	t-statistic	
Denmark (n = 45)	$rx\left(1-\frac{x}{K}\right)$	r = 0.5442 K = 4,896	4.252 -3.663[1]	R^2 = 0.1903 F = 9.8696
Iceland (n = 26)	$rx\left(1-\frac{x}{K}\right)$	r = 1.1008 K = 3669	6.325 -3.848	R^2=0.26 F = 14.8
Norway (n = 27)	$rx^2\left(1-\frac{x}{K}\right)$	r = 0.00021781 K = 8,293	5.51 18.22	R^2 = 0.62 F = 44.31

[1] The t-statistic is related to the b parameter in the estimated function g = aX + bX^2

Table 2.5 Parameter values and statistical properties of the demand functions. Capelin/Herring. Prices are measured in NOK/kg.

	Function	parameters	t-statistic	
Denmark (n=24)	$p(h) = a - bh$	a = 4.0104 b = 0.0007511	15.93 -10.70	R^2 = 0.7557 F = 61.8823
Iceland (n=12)	$p(h) = a - bh$	a = 1.211 b = 0.0001	14.83 -2.58	R^2 = 0.14 F = 5.43
Norway (n = 5)	$p(h) = 1$			

Table 2.6 Parameter values and statistical properties of the cost functions. Capelin/herring. Costs are measured in million NOK

	Function	parameters	t-statistic	
Denmark (n=10)	$C(h,x) = \alpha h^{1.33}$	α = 0.02198	15.4	R^2 = 0.6964
Iceland (n=219)	$C(h,x) = \alpha h^{2}$	α =0.000175	5.042	R^2 = 0.209 F = 33.35
Norway (n = 5)	$C(h,x) = \alpha h^{1.4}$	α = 0.07	32.12	R^2 = 0.98

3. Two Species Feedback Models

In this case biological interactions are taken into account. For Norway and Iceland the interaction between cod and capelin is modeled while for Denmark the interaction between Cod and Herring is modeled.

In general, the biological interdependent growth functions are:

$$\dot{x} = f(x, y) - h_x$$

$$\dot{y} = g(x, y) - h_y$$

The functional form used is:

$$f(x, y) = a_1 x^{\alpha} + b_1 x^{\beta} + c_1 xy$$
$$g(x, y) = a_2 y^{\sigma} + b_2 y^{\lambda} + c_2 xy$$

Where a_1, a_2, b_1, b_2, c_1 and c_2 are the parameters to be estimated and α, β, σ and λ are fixed coefficients. The results for each country are shown in table 3.1. - y is in all cases cod, while x is capelin for Norway and Iceland and herring in the case of Denmark.

Table 3.1 Parameter values and statistical properties of the multispecies biological functions. Growth is measured in 1000 tons.

	Function	Parameters	t-statistic	
Denmark (n=40)	$f(x,y)=a_1x+b_1x^2+c_1xy$	a_1 = 0.4351 b_1 = -6.476E-5 c_1 = -7.379E-5	4.772 -3.339 -0.7857	R^2 = 0.14
	$g(x,y)=a_2y+b_2y^2+c_2xy$	a_2 = 0.7007 b_2 = -0.0004745 c_2 = -2.902E-5	4.116 -2.577 -0.9402	R^2 = 0.21
Iceland (*n*=152)	$f(x,y)=a_1x+b_1x^2+c_1xy$	a_1 = 1.4734 b_1 = -0.0004 c_1 = -0.0004	5.6834 -4.6187 -1.8102	R^2 = 0.40
	$g(x,y)=a_2y+b_2y^2+c_2xy$	a_2 = 0.3518 b_2 = -0.0002 c_2 = 0.0001	2.9267 -2.1237 3.1298	R^2 = 0.42
Norway (n = 30)	$f(x,y)=a_1x^2+b_1x^3+c_1xy$	a_1 = 0.0018 b_1 = -1.19E-8 c_1 = -0.00021	4.9 -3.1 -3.4	R^2 = 0.59
	$g(x,y)=a_2y^2+b_2y^4+c_2xy$	a_2 = 0.00022 b_2 = -3.49E-11 c_2 = 1.82E-5	8.4 -4.2 2.6	R^2 = 0.50

It is assumed that there are no economic interactions and no interactions on the markets for fish, meaning that the profit for cod and capelin/herring fisheries can be added together, i.e. no need to estimate new demand and cost functions:

$$\pi(h_x, x, h_y, y) = p(h_x)\, h_x - C(x, h_x) + p(h_y)\, h_y - C(y, h_y)$$

4. Steady state stocks with and without harvesting

In this section we report the steady state stocks with and without harvesting in the deterministic model. The steady state stock shows the optimal long run equilibrium of the fishery in terms of size of harvest and of stock biomass.

Steady state stocks with Harvesting
We report the steady state stock and harvest figures for all species in all countries.

Denmark

	Stock (1000 tons)		Harvest (1000 tons)	
	Cod	Herring	Cod	Herring
Single-species	862	2,222	207	660
Multi-species	842	1,329	221	381

In the Danish competition model, two-species management implies lower standing stocks of both species, a bit higher cod harvest and significantly reduced herring harvest.

Iceland

	Stock (1000 tons)		Harvest (1000 tons)	
	Cod	**Capelin**	**Cod**	**Capelin**
Single-species	1,229	1,751	314	1,007
Multi-species	1,445	2,238	414	0

It is interesting to note that in the Icelandic predator-prey model the standing stocks of both species should be higher with two-dimensional modelling. The cod harvest is increased bu more that 30 percent whereas the capelin is not harvested at all in steady state. The surplus production of the capelin stock is entirely left in the ocean to feed the cod. This is in sharp contrast to the result from the single-species model.

Norway

	Stock (1000 tons)		Harvest (1000 tons)	
	Cod	Capelin	Cod	Capelin
Single-species	2,172	7,960	381	554
Multi-species	2,903	8,955	488	429

Also in the Norwegian predator-prey model the standing stocks of both species are higher. The harvest is increased for the predator, cod, and decreased for the prey, capelin, as part of the capelin surplus production is better used as feed for the cod.

Steady state stocks without harvesting

This is the two-dimensional equivalents of the carrying capacities. As the equations are highly non-linear, there are more than solutions for each country. Here the solutions with non-negative stock levels are reported.

Denmark

	Stock (1000 tons)	
	Cod	Herring
Single-species	1,433	4,984
Multi-species	1477	0
“	0	6,719
“	1,146	5,413

The first row shows the carrying capacities with the single species approach. The next two rows show the corresponding carrying capacities from the two species competition model when one the species has been eradicated. For cod it is seen that these two figures are fairly similar, it is only slightly higher when the competition from the herring has been eliminated. The herring stock, on the other hand, is significantly higher (35 percent) when the competition from the cod has been eliminated. Finally, the last row shows the case when both stocks are present and there is competition. As expected these are lower than when one stock is removed. For herring, however, it is higher than the carrying capacity in the single-species case.

Iceland

	Stock (1000 tons)	
	Cod	Capelin
Single-species	1,988	3,669
Multi-species	1,759	0
“	0	3,684
“	2,400	1,283

In the Icelandic case we have the same number of solutions as for Denmark, but the two-species approach is now based on a predator-prey model. For the cod this implies that the steady state without harvesting is lowest with the two-species model without the capelin to feed on and highest when there is an unharvested stock of capelin to feed on. For the capelin it is exactly the opposite, it highest when the predation pressure from the cod has been removed and lowest when there is an unharvested stock of cod. The single-species carrying capacities lay in between for both species.

Norway

	Stock (1000 tons)	
	Cod	**Capelin**
Single-species	2,473	8,293
Multi-species	2912	0
"	0	15,126
"	3,078	5,866
"	3,153	8,814

The Norwegian case is a bit different as there is one more steady state to analyse. The steady state with the lowest stock levels is, however, only semi-stable and can therefore be ignored for practical purposes. It is the one with the highest stock levels (bottom row) that would eventually come into existence if both stocks were left unharvested for a long time. This case yields the highest cod stock whereas the capelin stock could be much higher if the predator, the cod, was removed. Notice, however, that both stocks are higher with the two species approach than with the single-species approach in the non-trivial stable steady state.

5. Evaluation of fishery policies

Having completed the construction of our simple fisheries model we are now in a position to assess the relative efficiency of the cod harvesting policies followed by the three countries in the past. For this purpose we employ two main criteria; (i) the "economic health" of the cod stock measuring by the degree of stock overexploitation and (ii) the "appropriateness" of the annual harvest where while the degree of overharvesting is measured. The former is measured by the actual stock size relative the optimal steady state level. The latter is measured by the actual annual harvest relative to the optimal one.

Comparative Stock evaluation

Here we look at the parameter η which measures the degree of stock overexploitation. This parameter is defined as

$$\bar{\eta} = \frac{1}{n}\sum_t \eta_t = \frac{1}{n}\sum_t \frac{x^t_{act}}{x^*} = \frac{\sum x^t_{act}}{\sum x^*}$$

Where x^t_{act} is the actual stock in period t and x^* is the optimal long-term steady state stock. Note that $\bar{\eta} < 1$ represents stock overexploitation whereas $\bar{\eta} > 1$ represents underexploitation.

Denmark

	Cod	Herring
Single-species	0.59	1.12
Multi-species	0.61	1.88

This confirms the result from the harvest evaluation that Danish herring is underexploited both in the single-species and the multi-species model whereas Danish cod is overexploited. Due to the competition aspect of this model, the optimal stock level is lower for both species when the multi-species approach is being used, and this makes $\bar{\eta}$ larger.

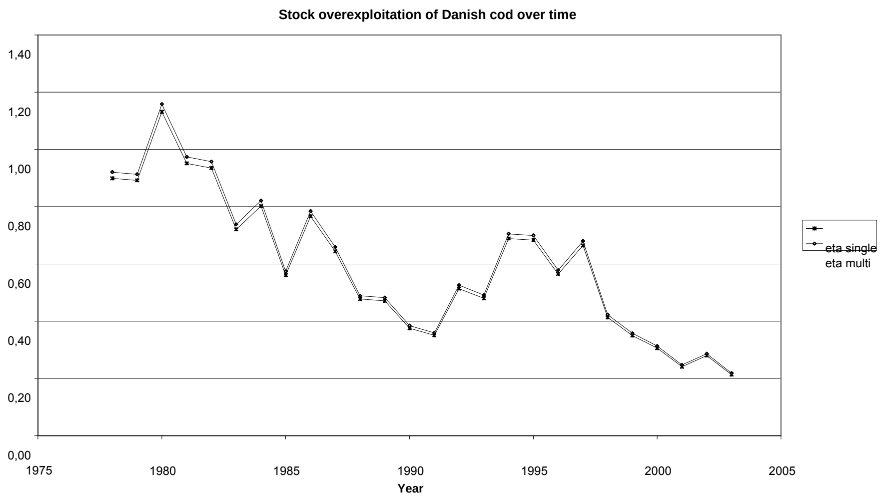


Figure 5.1 Stock overexploitation of cod over time

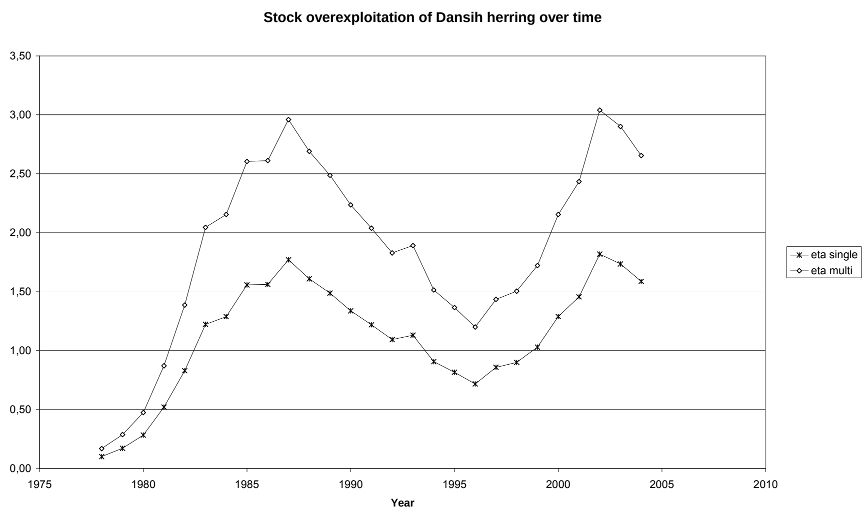


Figure 5.2 Stock overexploitation of herring over time

Iceland

	Cod	Capelin
Single-species	0.53	1.22
Multi-species	0.43	0.88

The Icelandic cod stock is overexploited both in the single-species and the multi-species model. And also the stock-exploitation parameter indicates higher overexploitation with the two-species approach. The capelin stock, on the other hand, seems to be underexploited in the single-species model but overexploited in the multi-species model. This is also in line with the result from the harvest overexploitation parameter. In other words, the two-species approach calls for a more conservative exploitation pattern of both species when the two-species approach is applied.

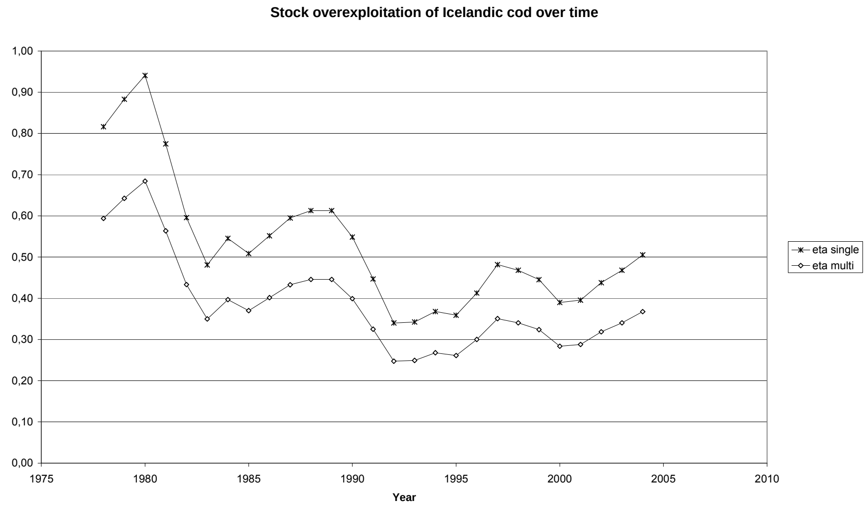


Figure 5.3 Stock overexploitation of cod over time

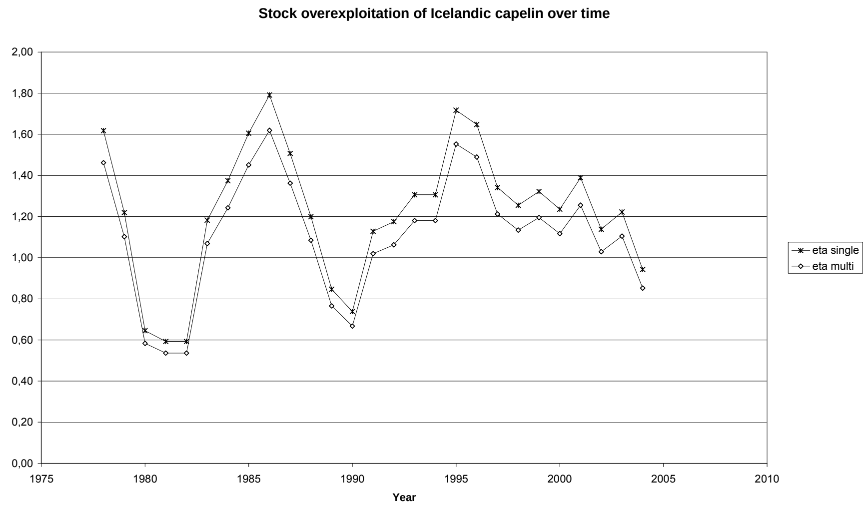


Figure 5.4 Stock overexploitation of capelin over time

Norway

	Cod	Capelin
Single-species	0.61	0.35
Multi-species	0.46	0.31

Both the Norwegian cod stock and the capelin stock is severely overexploited both in the single- and multi-species model. Capelin is more overexploited than cod, and the degree of overexploitation is higher in the multispecies model than in the single-species as the optimal stock level for both species is higher in the multi-species model.

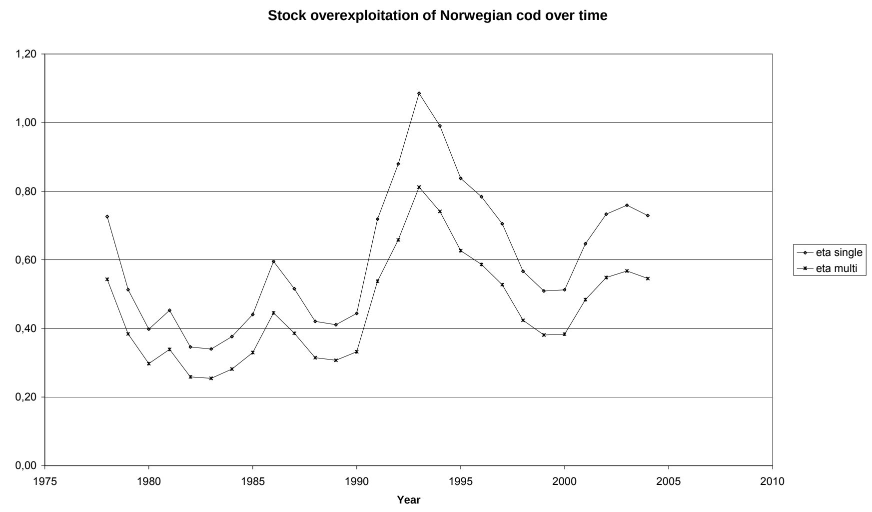


Figure 5.5 Stock overexploitation of cod over time

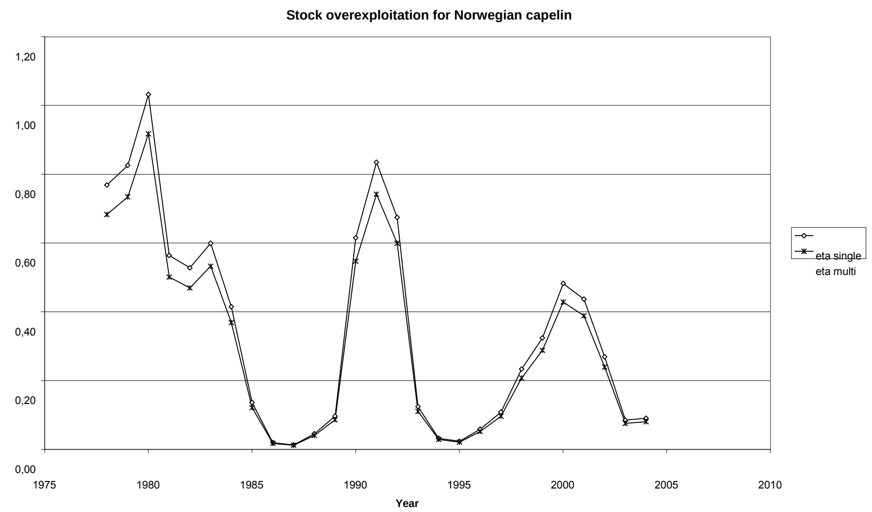


Figure 5.6 Stock overexploitation of capelin over time

Comparative harvest evaluation

Here we look at the parameter φ which is supposed to measure the degree of overharvesting. This parameter is defined as

$$\varphi = \frac{\sum h_{act}}{\sum h_{opt}}$$

Where h_{act} is the actual harvest and h_{opt} is the optimal harvest. Note that $\varphi > 1$ represents overharvesting whereas $\varphi < 1$ represents underharvesting.

Denmark

	Cod	Herring
Single-species	4.15	0.89
Multi-species	3.80	0.62

It is interesting to note that Danish herring seems to be underexploited both in the single-species and the multi-species model. Optimal harvest is higher for both species when the multi-species approach is being used, and this makes φ smaller. This is probably an implication of the competition between the species.

Iceland

	Cod	Capelin
Single-species	11.80	0.83
Multi-species	16.24	4.79

Notice that there is a very high degree of overexploitation of cod in Iceland. The value of φ is higher with the single-species approach than with the two-species approach. The reason for this is that the optimal standing stock is higher with the two-species approach, and it is therefore necessary to reduce the harvest pressure in order to let the stock build up to this level.

It is interesting to note that φ for capelin is not only larger with the two-species approach meaning that optimal harvest is smaller, but the indicator goes from indicating harvest underexploitation to harvest overexploitation when the two-species approach is applied. The reason for this is that capelin has an alternative use as food for the cod with this approach. Hence the standing stocks of both species are higher with the two-species approach. The two-species approach implies, in other words a more conservative optimal management regime not only for capelin but for cod as well.

Norway

	Cod	Capelin
Single-species	3.42	2.24
Multi-species	3.56	3.71

Also in the Norwegian case it is seen that the difference between the single-species and the multi-species approach is not very large for cod. And, as in the case of Iceland, φ for capelin is larger with the multi-species approach for the same reason.

6. Discussion about the results

One of the purposes of using different models is to get information about the relative merits of the models and on whether more complicated models yield better results. Therefore, the results from the deterministic single and multispecies models and from the stochastic single species model are compared country by country.

6.1 Discussion about the Norwegian results

Cod: results from the single and multi-species models

Figure 6.1 illustrates the optimal feedback curves for cod based both on deterministic and stochastic modelling together with the surplus growth curve and actual harvest. The upper red curve represents static optimization that is maximizing net revenue at each point in time given the present stock level without considering the future. This is the optimal policy for a sole owner who is completely myopic, also called open access equilibrium. The other optimal feedback curves are all calculated with five percent discounting and different levels of stochasticity. The upper one (black) is the optimal deterministic policy, whereas the other two are calculated for $\sigma(y) = 0.1y$ and $\sigma(y) = 0.5y$, respectively. The latter one represents the case of a fairly high degree of stochasticity. Nevertheless, it is seen that these curves stay so close together that they for practical purposes can be regarded as a single curve. The conclusion therefore is that stochasticity does not affect the optimal policy as long as we use reasonable levels of stochasticity. Note also that the actual harvest is far above the optimal harvest and is probably the result of a policy aiming at maximum sustainable yield.

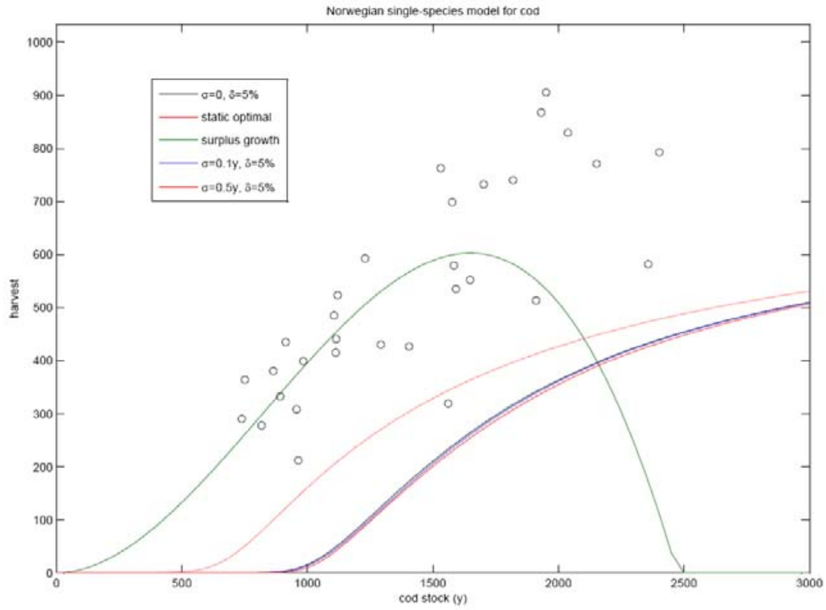


Figure 6.1 Norwegian single-species model for cod. Harvest and growth is 1000 tons.

Figure 6.2 illustrates the same results and the same pattern in time space. The upper red curve represents actual harvest whereas the optimal feedback curves with five percent discounting and various degrees of stochasticity again are clustered together and these are hard to distinguish from the deterministic optimum. It is interesting to note, however, that the actual harvest sometimes is lagged compared with the optimal harvest. This indicates that if the optimization model had been used, the necessary changes in policy would have taken place earlier and this might have stabilized the stock. The thick green curve, representing myopic optimization, lies a bit above the rest, and the thick blue curve represents the optimal cod policy when two-species interaction with capelin is taken into account. Optimal harvest based on multi-species modelling also shows the same pattern except in the late 90s and early 2000s. Here some extra harvest of cod is necessary in order to save the capelin. This will be further discussed in the next paragraph.

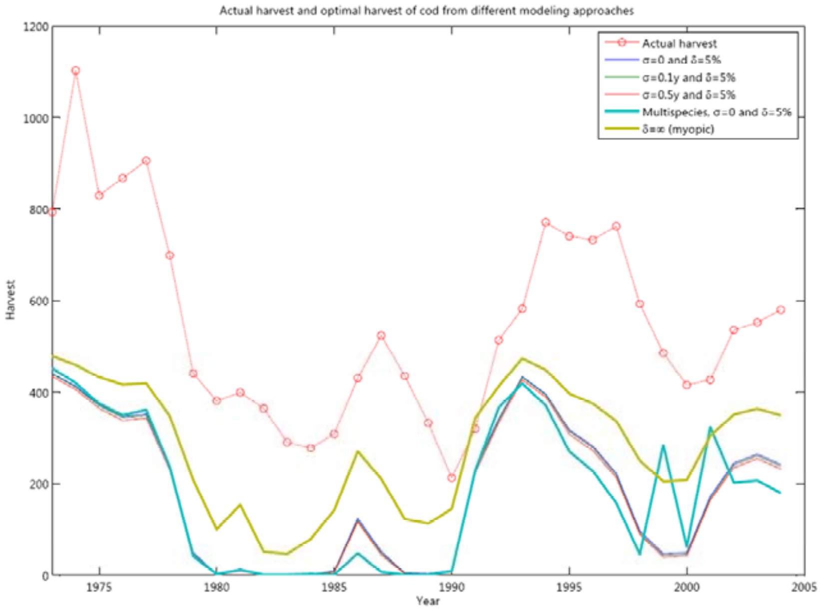


Figure 6.2 Actual harvest and optimal harvest of cod from different modeling approaches (1000 tons).

The optimal cod policy in a multi-species perspective is further visualized in Figure 6.3. Here we can see the optimal harvest of cod for various combinations of the cod- and capelin stock. Notice that in most part of this three-dimensional diagram the harvest of cod is virtually unaffected by the capelin stock; it is more or less the two-dimensional curve projected into three dimensions. However, for a certain combination of cod- and capelin stocks, a peak emerges in the diagram indicating that the cod harvest ought to much higher in this particular area. The reason for this is that the addition of a multi-species interaction term in the growth equation for capelin induces critical depensation. Critical depensation means that there is a lower critical biomass below which the capelin stock will go extinct even without harvesting. By putting extra effort into cod harvesting in this case, the area of critical depensation will be reduced and extinction may be avoided. It is only for a relatively small area of combinations of the cod and capelin stock that this extended effort is in effect. The smaller the capelin stock, the smaller the cod stock will be where extended effort is needed.

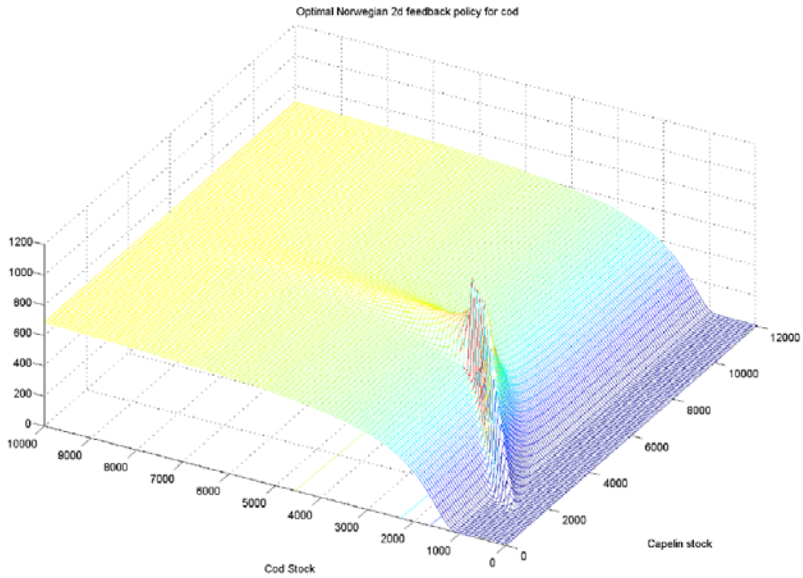


Figure 6.3 Optimal Norwegian 2d feedback policy for cod (1000 tons)

Capelin: results from the single and multi-species models

Figure 6.4 illustrates optimal feedback curves for capelin harvest based on a single-species model with various degrees of stochasticity, namely $\sigma(x) = 0.1x$ and $\sigma(x) = 0.5x$. The surplus growth function and actual historical harvest are also depicted in this figure. All the optimal harvest paths are calculated with five percent discounting. As the revenue function is independent of the stock, the static optimum (bliss) is constant in this diagram. For larger stock levels, all optimal paths approach the static optimum. In particular, this can be seen for stock sizes above the msy-stock size. For stock levels below one million tons all paths indicate harvest moratorium. The difference between the paths occurs between one million tons and the msy stock which is 5.5 million tons. In the deterministic case harvest increases sharply from the moratorium level and coincide with the static bliss very early whereas in the case with highest stochasticity harvest is more conservative and approach the static level only gradually.

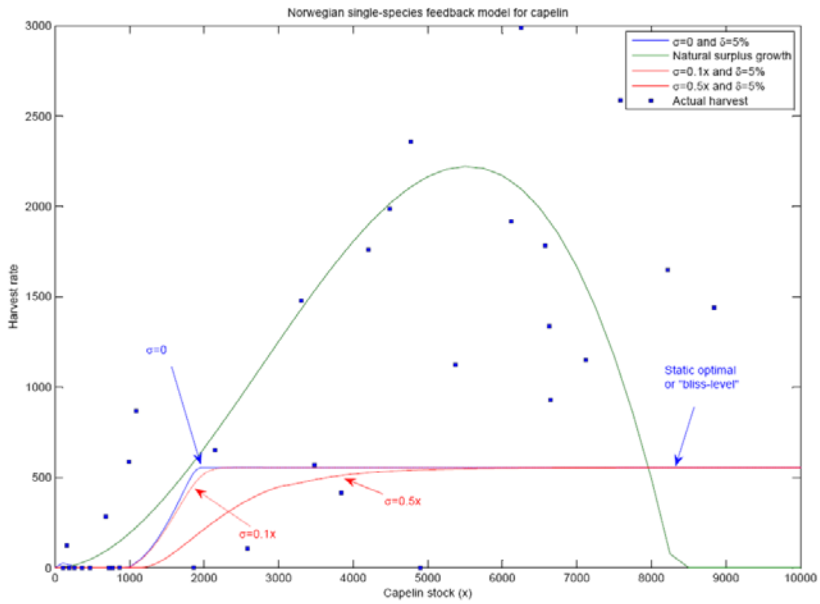


Figure 6.4 Norwegian single-species feedback model for capelin (1000 tons)

The time paths for the same levels of stochasticity together with the optimal path based on multi-species modelling are illustrated in Figure 6.5. Actual harvest is also shown in this figure and is seen to be high above the optimal for long periods. The single-species stochastic paths seem to stick fairly close together with the highest degree of stochasticity implying the most conservative harvest as expected. The optimal path based on multi-species modelling is a bit different. For most of the time this path is more conservative than the single-species paths except in a few periods when the single-species model suggests harvest moratorium.

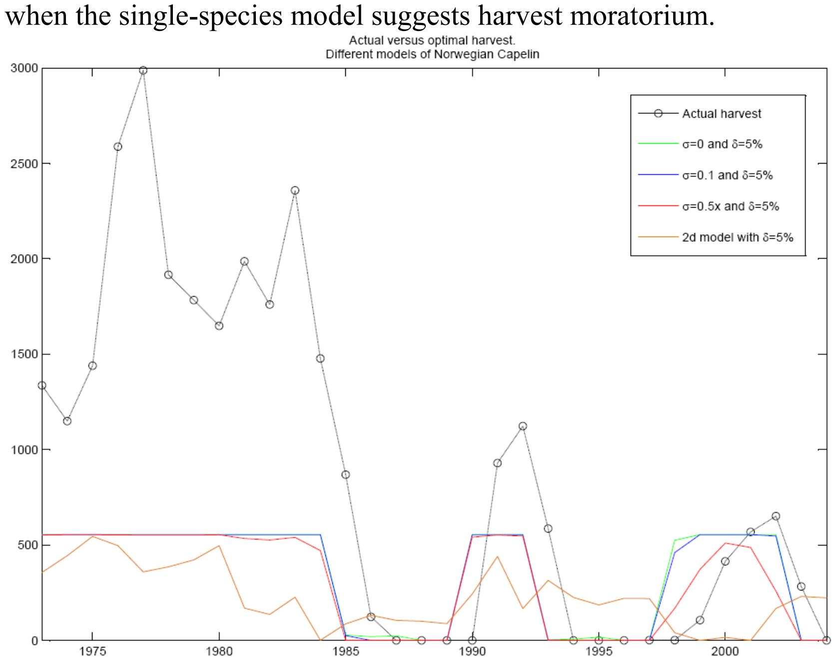


Figure 6.5 Actual versus optimal harvest. Different models of Norwegian capelin. (1000 tons)

Figure 6.6 shows the optimal capelin harvest in the two-dimensional cod- and capelin-stock space. For very small cod levels the optimal harvest plane for capelin is similar to the single-species path, namely a steep rise from the moratorium to the static bliss level. For larger cod stock levels a quite interesting patterns emerges. This pattern consists of considerable harvest for low capelin stocks, then a moratorium over a certain range and then a gradual approach to the static optimum for higher stock levels. It is in particular the high harvest at low stock levels that is intriguing because it seems somewhat counterintuitive. The reason why it should be so is that the presence of the cod stock in this model induces critical dispensation. In other words, there is a lower critical biomass of capelin below which the stock inevitably goes extinct even without harvesting, and it is therefore no reason to restrict harvesting in this area. But, as we saw in Figure 3, it is possible to reduce this area by increasing the cod harvest.

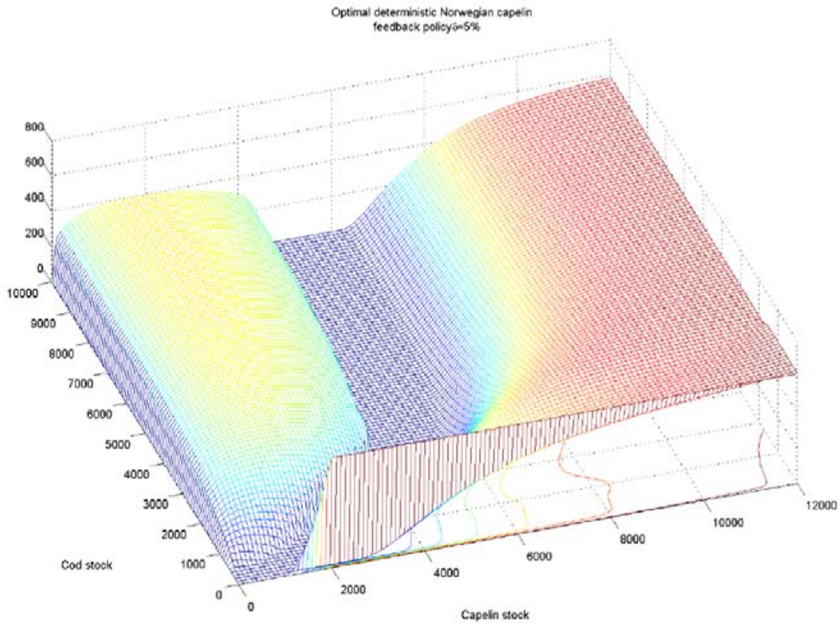


Figure 6.6 Optimal deterministic Norwegian capelin. Harvest = 1000 tons.

Discussion about actual harvest

Actual harvest of cod compared to the optimal harvest from the two-dimensional model has been higher for the total period we are looking at, see Figure 6.7. Particularly in the period before 1990, when the two-dimensional model for a large part advocated harvest moratorium, the actual harvest was high. For a few years in the early 90s, especially 1991 – 1993 the difference between actual and optimal was reasonable although there was a difference. In these years Norwegian managers bragged about being world champions in cod management, and the biomass increased. Unfortunately, from the mid-90s Norwegian managers reverted to the old pattern of overexploitation and it seems that this still is going on.

The actual harvest of capelin has switched from high harvest to periods with harvest moratorium, see Figure 6.8. The two-dimensional model, on the other hand, has advocated a more even harvest pattern over the period varying between zero and 500,000 tons. If the optimal pattern had been followed the upper harvest could have been even higher. It is interesting to note that the periods with actual harvest moratorium has not been the same as the periods suggested by the model. As late as 2004 there was an actual moratorium whereas the model suggested a harvest of some 220,000 tons. In 2001, on the other hand, the model suggested moratorium whereas actual harvest was close to 570,000 tons. In periods actual and optimal harvest has in fact been a bit countercyclical, revealing that there has been no sign of multi-species considerations in the actual management; at least not of the kind suggested here.

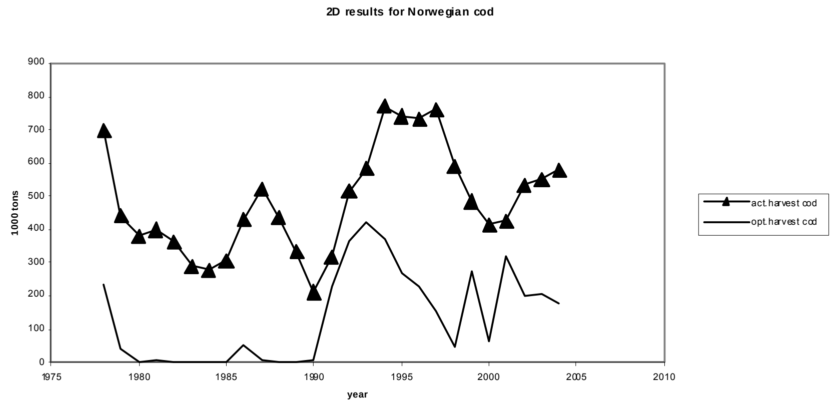


Figure 6.7 Actual harvest of cod compared to optimal harvest based on the two-species model

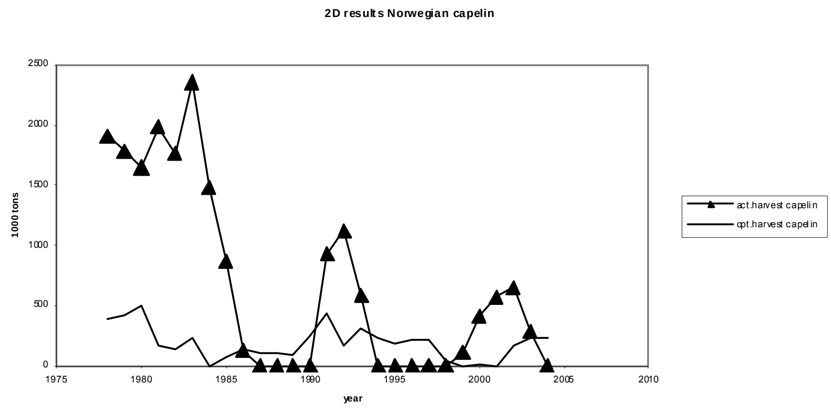


Figure 6.8 Actual harvest of Capelin compared to optimal harvest based on two-species model

6.2 Discussion about the Icelandic results

The Icelandic study dealt with two species, cod and capelin. Cod, it is well known, preys on capelin, which constitutes an important part of the cod's diet (Jakobsson and Stefansson 1998, Marine Research Institute 2006). Estimates of the biomass growth functions, reported in some detail in the Appendix, resulted in the following equations:

$$\dot{y} = 0.3518 \cdot y - 0.0002 \cdot y^2 + 0.0001 \cdot y \cdot x\,,$$
$$\dot{x} = 1.4734 \cdot x - 0.0004 \cdot x^2 - 0.0004 \cdot x \cdot y\,,$$

Where y represents the biomass of cod and x that of capelin.

Both stock interaction parameters exhibit the expected sign. The one for the impact of capelin on cod proved strongly significant (t-statistic = 3.1). The one describing the impact of cod on capelin was just barely significant (t-statistic = 1.8). The impact of capelin on cod can be very substantial in terms of the cod's biomass growth. Thus, at its average size (during the sample period) the capelin stock this term adds about 0.17 or almost 50% to the intrinsic growth rate of the cod. This increases the virgin stock equilibrium and the maximum sustainable yield of cod very substantially compared to the situation where there is no capelin. The negative impact of cod on the biomass growth of capelin appears less. At its average size (during the sample period) the cod stock reduces the intrinsic growth rate of capelin by 0.28 or about 19% compared to the situation where there is no cod.

The following figures provide sustainable yield diagrams for cod and capelin. Three diagrams are given for each species corresponding to three stock sizes of the other species. More precisely, these three sustainable yield diagrams correspond to (i) the maximum stock size and (ii) the average stock size of the other species during the data period and (iii) zero stock size of the other species.

Figure 6.9
Cod: sustainable yield

(Solid line: Average capelin stock
Dotted line: Maximum capelin stock
Dashed line: Zero capelin stock)

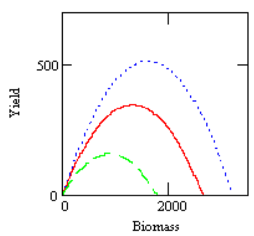
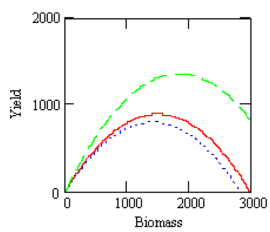


Figure 6.10
Capelin: sustaianble yield

(Solid line: Average cod stock
Dotted line: Maximum cod stock
Dashed line: Zero cod stock)



The following figure provides aggregate sustainable yield contour diagrams (equiyield diagrams) for the two species in biomass space. More precisely, these diagrams draw contours for the function:

$$10\cdot h_{cod} + h_{capelin} = 10\cdot \dot{y} + \dot{x} \qquad (3)$$

Where $\dot{y}$ and $\dot{x}$ are as defined in equations (1) and (2). The multiplication by the factor 10 is to reflect the great difference in the unit value of cod vs. that of capelin. In the first diagram, no species interactions are assumed. In the second the estimated interactions (equations (1) and (2) above) are adopted.

Figure 6.11

Yield contour diagram: No species interctions

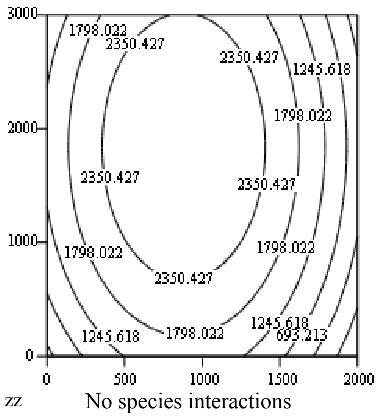
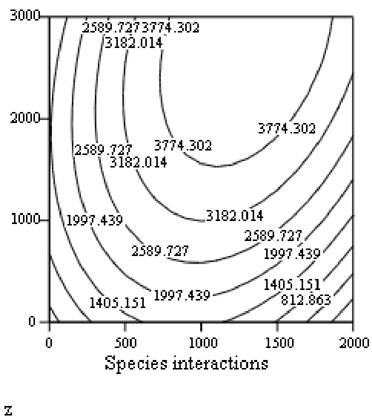


Figure 6.12

Yield contour diagram: Species interctions



A glance at the diagrams in figures 6.11 and 6.12 shows that estimated species interactions has a substantial effect on the sustainable yields and therefore, presumably, the optimal harvesting paths of the two species. In other words, it would entail significant errors to separately manage the cod and capelin stocks, if the true interactions are as in equations (1) and (2) and depicted in Figures 6.10 and 6.12.

Given the above specifications, i.e. equations (1) and (2) and the stochastic specifications in a previous chapter, profit maximizing feed-back harvesting paths for cod and capelin have been worked out. Let us first look at the species singly, i.e. without the species interactions.

6.2.1 Optimal harvesting policies: No species interactions

Cod

The following Figure 6.13 illustrates the optimal feed-back paths for cod for varying volatility parameters, σ. Feed-back policies for the following three volatility parameters have been calculated:

σ=0, i.e. the nonstochastic case
σ=0.1·*y*
σ=0.5·*y*,

Where, as before, *y* represents the biomass of the cod stock. For comparison purposes we also draw in Figure 6.13, the zero marginal profit schedule which corresponds to unmanaged fishing (referred to as 'static optimal' in the diagram) and the actually observed harvest biomass coordinates. Note that these have occurred over a period of over 20 years and therefore apply partially to a different technology and prices.

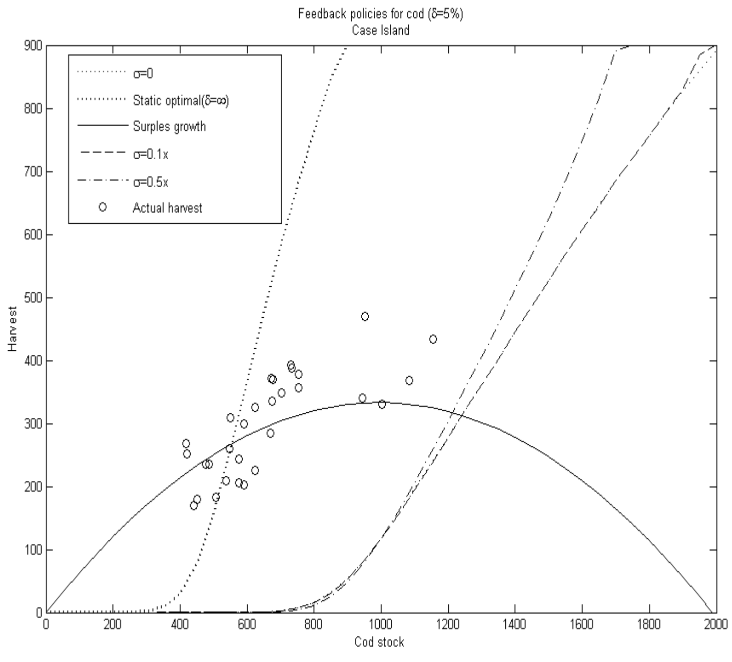


Figure 6.13 Cod: Optimal feed-back harvesting. No species interactions. Harvest = 1000 tons.

The following observations are readily made:

- All the optimal feed back paths are very conservative compared to open access fishing (and the experience). Harvesting should cease completely for a cod stock below 700.000 metric tonne, — a stock larger than in most years in the data set. The optimal sustainable equilibrium occurs at a biomass level of just over 1200.000 metric tonne and harvest rate of some 300.000 metric tonne.

- There is little difference between the optimal paths for different stochastic specification if the biomass level is relatively low. However, at large stock sizes, the difference between the paths becomes substantial. This is no doubt a consequence of the volatility parameter being proportional to the stock size.
- At comparatively very low levels of biomass, between 700.000 and 1000.000 metric tonne, say, there are signs that higher volatility (greater biomass growth uncertainty) leads to more conservative harvesting. This effect, however, reverses itself at higher stock levels. Again, this appears intuitive. Due to the mean reverting nature of the stochastic biomass growth process, there is a much greater chance of a negative stock movement when the stock is large, so it is a good idea to reduce the uncertainty. At low stock levels this argument is simply reversed.
- None of the actual biomass-harvest co-ordinates are anywhere close to what is found to be dynamically optimal. The all represent hugely excessive harvesting at the existing biomass levels.
- Interestingly, according to the 'static optimal' curve, the fishery might be profitable down to biomass level of some 300.000 mt less than a quarter of the optimal sustainable biomass level.

In Figure 6.14, we draw the optimal feed back harvesting programs according to the actual biomass levels each year since 1975 and compare this with the actual harvest. Two optimal paths for no uncertainty (σ=0) are drawn. One is the single species optimal, labeled '1d-feedback'. The other takes species interactions into account, labeled '2d-optimal'. As evident from the diagram, the optimal harvest has almost always been zero in this period and every year the actual harvest has been greatly excessive.

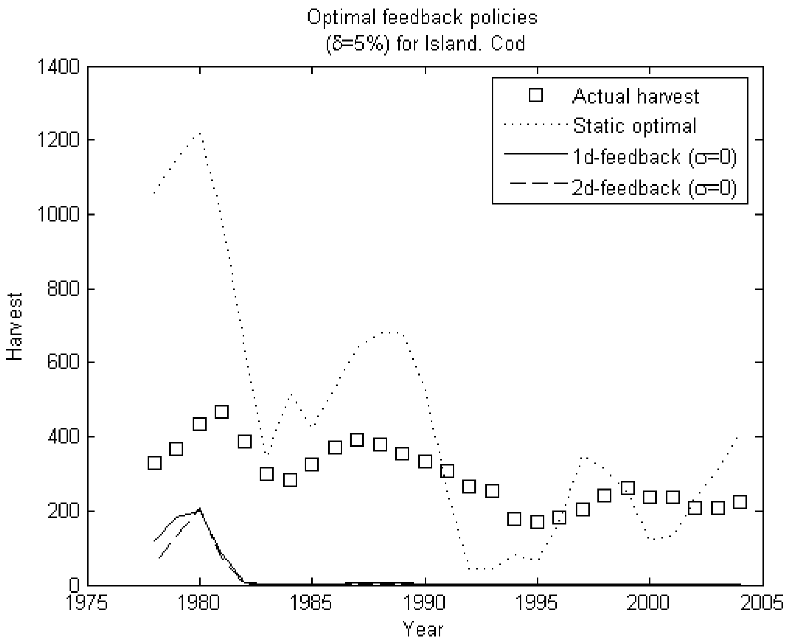


Figure 6.14 Cod: Actual and optimal harvest. Harvest = 1000 tons.

Capelin

The optimal feed-back policies for capelin at same levels of the volatility parameter as before, namely:

σ=0, i.e. the nonstochastic case

$\sigma=0.1 \cdot x$

$\sigma=0.5\ x$,

Where x refers to the biomass of capelin. For comparison purposes we also draw in Figure 6.15, the zero marginal profit schedule which corresponds to unmanaged fishing (referred to as 'static optimal' in the diagram) and the actually observed harvest biomass co-ordinates.

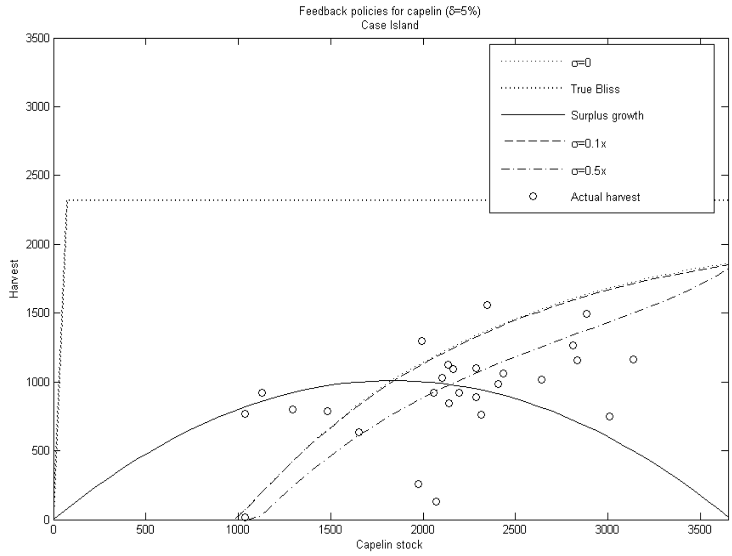


Figure 6.15 Capelin: Optimal feed-back harvesting policies. No species interactions. Harvest = 1000 tons.

The inferences we can draw from Figure 6.15 are somewhat different from those for the cod above.

- The optimal feed-back paths are not particularly conservative compared to the actually observed fishing. Since the open access harvesting is much higher, this must be because of the quite restrictive TAC-policy employed in the capelin fishery virtually from the outset.
- There is significant difference between the optimal paths for different stochastic specification. The high risk situation (σ=0.5) leads to substantially more conservative harvesting policies at all levels of biomass than the riskless and low risk situations (σ=0, σ=0.1). On the other hand there is little difference in the optimal paths for the riskless and low risk situations.
- The actual biomass-harvest co-ordinates are distributed around the optimal path, but not particularly close to it. If anything the actual harvest seems to more often suboptimal rather than excessive.

In Figure 6.16, we draw the optimal feed back harvesting programs according to the actual biomass levels each year since 1978 and compare this with the actual harvest. Two optimal paths for no uncertainty (σ=0) are drawn. One is the single species optimal, labeled '1d-feedback'. The other takes species interactions into account, labeled '2d-optimal'.

As evident from the diagram, the actual harvest is distributed around the single species optimal one. This suggests that the actual capelin harvesting policy since 1978 has been in the neighbourhood of the optimal policy. However, it has probably not been very close to the optimal policy. Annual deviations from the calculated optimal policy are too great to make that a reasonable assumption, even allowing for inaccuracies in the calculation of the optimal policy.

Taking the interaction of the capelin with the cod stock into account leads to the 2d-optimal capelin harvesting policy (dashed curve). This represents much lower capelin catch every year. The reason, of course, is that according to our estimates, capelin constitutes important feed for cod. Compared with this two-species optimal harvesting policy, the actual capelin harvest has been excessive in most years.

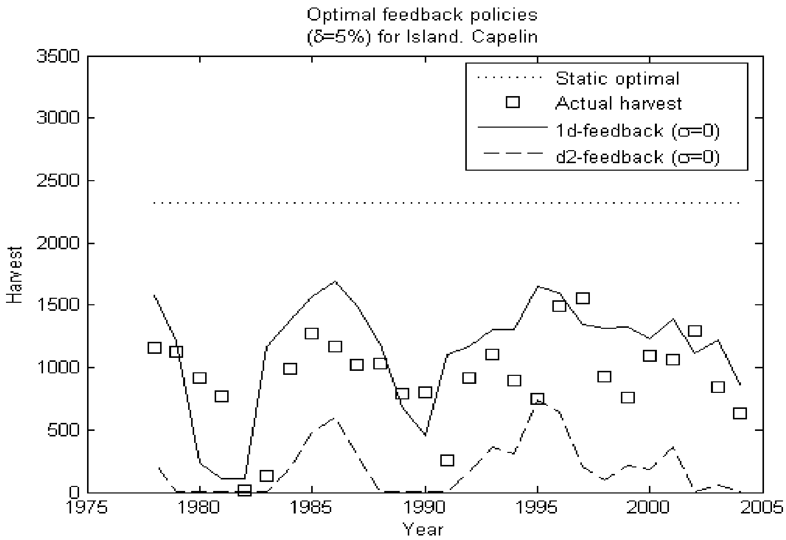


Figure 6.16 Capelin: Actual and optimal harvesting policies. Harvest = 1000 tons.

Optimal harvesting policies: Species interactions

Under species interactions, the optimal harvest policy of one species depends on the stock size of the other species. Harvest feed-back diagrams, therefore, need to be three dimensional.

The following two diagrams provide feed-back diagrams for cod and capelin, respectively. Figure 6.17 illustrates the optimal feed-back policy for cod. As shown in the diagram, there should be no harvesting of cod unless its biomass is excess of 500.000 metric tonne. The size of the capelin stock has little effect on this. The minimum biomass before harvesting should begin increases slightly with the biomass of capelin. A possible explanation is that when the biomass of capelin increases the intrinsic growth rate of cod increases and thus it is more beneficial to conserve it. The same effect can be seen at higher cod biomass levels: harvest is generally slightly lower the –bigger the stock of capelin. However, at very low stock levels of capelin this effect is reversed, probably to save the capelin.

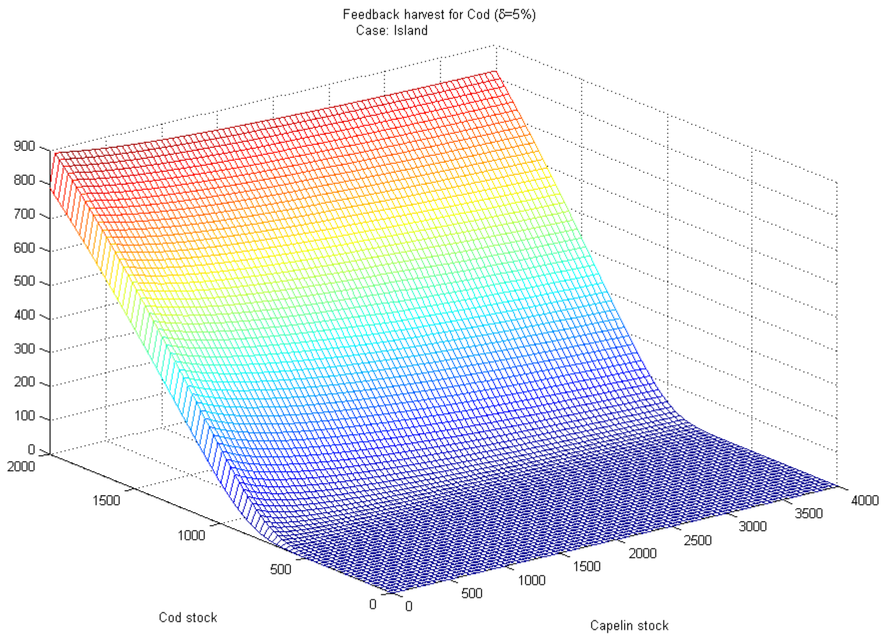


Figure 6.17 Cod feed-back harvesting policies. Stock and harvest = 1000 tons.

Since 1995, a catch-rule has been in effect in the cod fisheries, which stipulates that each fishing year's TAC should equal 25% of the fishable stock. This simple rule of thumb is, however, not optimal, as catches will be too high when stocks are low, and too low when stocks are high. In the years since the rule was introduced, the cod stock has hovered between 450 and 600 thousand years, and catches varied between 180 and 260 thousand tons. The discrepancy between the rule and catches illustrates the fact that the rule has not been completely adhered to. However, these catches are far greater than optimal.

The capelin harvesting feed-back diagram is more complicated. Capelin should not be harvested at all until it reaches about 1400.000 Metric tonnes. From then on the harvesting decreases fast with the size of the cod stock and therefore its need for capelin feed.

Capelin catches have also far exceeded the optimal feedback harvesting policy. As shown in Figure 6.18, actual harvest has been close to the single species optimum, but when the interaction with cod is also taken into account, it becomes clear that capelin has been overfished.

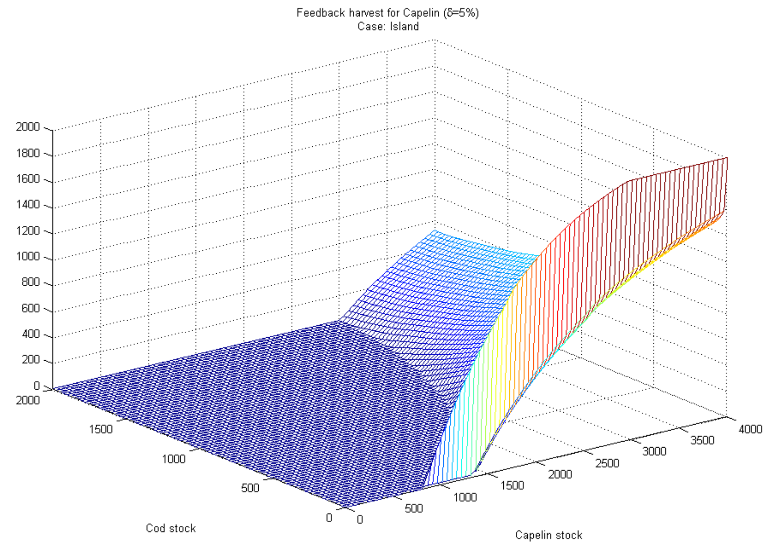


Figure 6.18 Capelin feed-back harvesting policies. Stock and harvest = 1000 tons.

The following phase diagram in biomass space further illustrates the optimal dynamic paths for the biomass of cod and capelin from any initial position. Four equilibria exist, but only one of them, located at roughly (cod=1.440.000 Mt, capelin=2.200.000 Mt), is stable. In fact it seems to be globally stable, provided both initial biomasses are positive. At this equilibrium, there will be no harvest of capelin. The stock is used exclusively as food for cod.

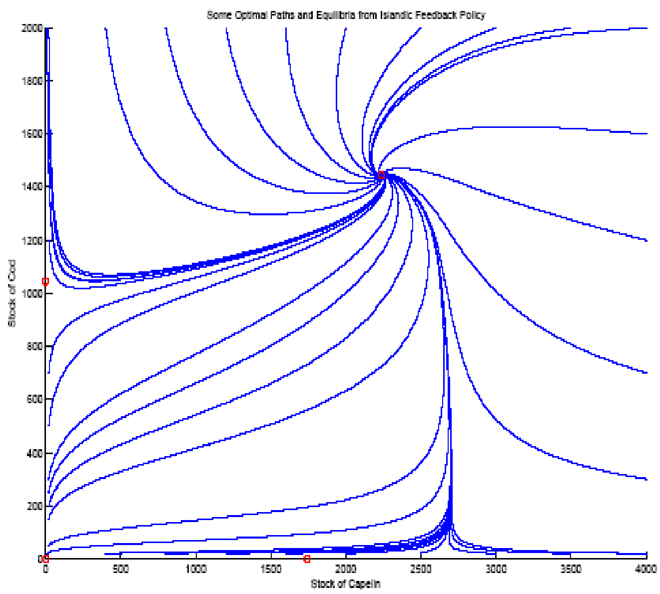


Figure 6.19 Cod-capelin biomass: Optimal phase diagram. 1000 tons.

6.3 Discussion about the Danish results

Estimates of the biomass growth functions, reported in some detail in the Appendix, resulted in the following equations:

$$\dot{y} = 0.7007y - 0.0005y^2 - 0.0003x\,,$$

$$\dot{x} = 0.4351x - 0.0006x^2 - 0.0007y\,,$$

Where *y* represents the biomass of cod and *x* that of herring.

The negative signs of the interaction parameters indicate that the species are competitors for the same resource. All things equal, there is a negative impact of the other species on the biomass growth of the first species. This reduces the sustainable yield of each species compared to a situation where there is no interaction. However, these terms are not significant (t-statistic = -0.9 and -0.7). So the conclusion is that the interaction or interdependency between cod and herring in the North Sea can be rejected by this two-species model.

In the following, we will, however, present the result of using both the single species models and the two-species model.

Single species model: Cod
The figure 6.20 shows the optimal feed-back paths for cod for varying volatility parameters, σ. Feed-back policies for the following three volatility parameters have been calculated:

- $\sigma=0$, i.e. the nonstochastic case
- $\sigma=0.1\cdot y$
- $\sigma=0.5\cdot y$,

Where, as before, *y* represents the biomass of the cod stock. For comparison purposes the zero marginal profit schedule which corresponds to unmanaged fishing (referred to as ‘static optimal’ in the diagram) and the actually observed harvest biomass co-ordinates are shown as well. Finally the surplus growth schedule is drawn.

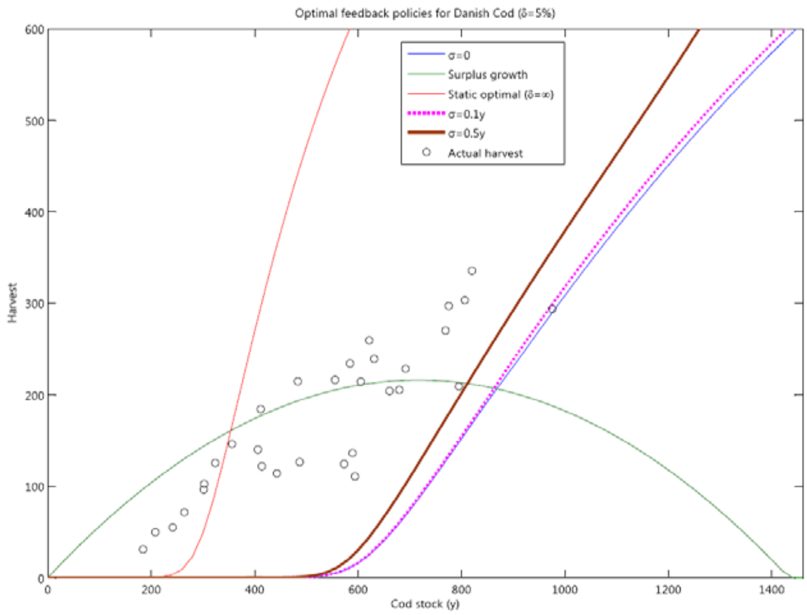


Figure 6.20 Optimal feedback polities for cod. No species interaction. 1000 tons.

The following observations can be made. All the optimal feed back paths are very conservative compared to open access fishing (and the experience). Harvesting should cease completely for a cod stock below 500.000 metric tonne. The optimal sustainable equilibrium occurs at a biomass level of 800.000 metric tonne and harvest rate of some 200.000 metric tonne. There is a very little difference between the optimal paths for the non-stochastic and lower volatility parameter cases. When the volatility parameter is higher the optimal path becomes different - about 20% higher harvests for a given stock size. None of the actual biomass-harvest observations are anywhere close to what is found to be dynamically optimal. The all represent excessive harvesting at the existing biomass levels. However, according to the 'static optimal' curve, the fishery might be profitable down to biomass level of some 200.000 mt - a quarter of the optimal sustainable biomass level, indicating why the fishery continues.

The next figure 6.21 shows the same results now in a time frame. The feedback policy with higher volatility produces significantly higher harvest-levels than the deterministic and lower volatility feedback policy and interesting the higher harvest level corresponds to the two-species feedback policy. This will be discussed further in the next paragraph. The actual harvest expect for one year much higher than the harvest levels produced by the optimal feedback policies. In fact except for 3 years since 1998, the optimal feedback policy - given the stock sizes in those years - was to close the fishery.

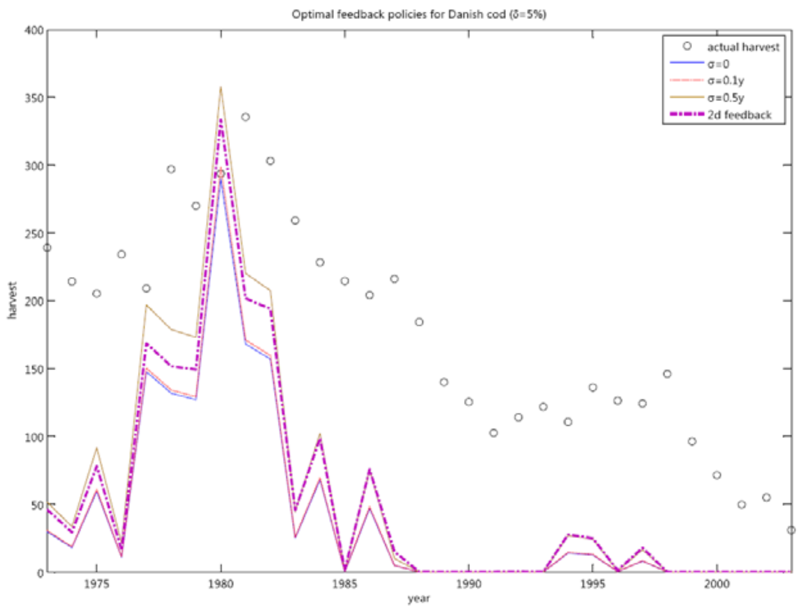


Figure 6.21 Optimal feedback harvest polities for cod (1000 tons).

Herring

The optimal feed-back policies for herring at same levels of the volatility parameter as before, namely:

σ=0, i.e. the no stochastic case

σ=0.1·*x*

σ=0.5 *x*,

where *x* refers to the biomass of herring. For comparison purposes we also draw in Figure 6.22, the zero marginal profit schedule which corresponds to unmanaged fishing (referred to as 'static optimal' in the diagram) and the actually observed harvest biomass co-ordinates.

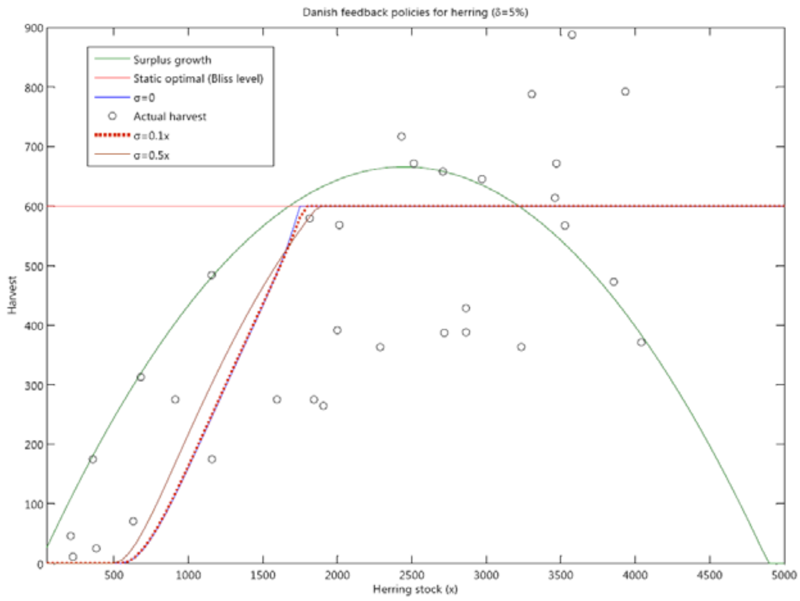


Figure 6.22 Optimal feedback polities for herring. No species interaction. 1000 tons.

The inferences we can draw from Figure 6.22 are somewhat different from those for the cod above. All three optimal feedback polities are very similar, so stochasticty does not change the conclusion. The optimal feedback paths are not particularly conservative compared to the actually observed fishing. The actual biomass-harvest co-ordinates are distributed around the optimal path, but not particularly close to it. In fact, the actual harvest seems to more often suboptimal rather than excessive. This has been the case since 1993. The optimal feedback paths indicate a very simple harvest rule. If the stock is less than around 600.000 metric tonne the optimal policy is to close the fishery and if the stock size is above 1700.000 metric tonne, the harvest level is constant, namely 600.000 metric tonne. If the stock size is between 600.000 and 1.700.000 metric tonne, the harvest can be increased by around 0.5 kg per kilo stock biomass increase, e.g. if the stock biomass is 1.000.000 metric tonne then the optimal harvest is 200.000 metric tonne.

In Figure 6.23, we draw the optimal feedback harvesting programs according to the actual biomass levels each year since 1973 and compare this with the actual harvest. Two optimal paths for no uncertainty (σ=0) are drawn. One is the single species optimal, labeled σ=0. The other takes species interactions into account, labeled ‘2d-feedback’. The actual policy has until 1985 been delayed compared to the optimal feedback policy. After 1985 the actual harvest has been above the optimal level until 1993 and below thereafter. However, the actual harvest has in the recent years been approaching the optimal harvest level.

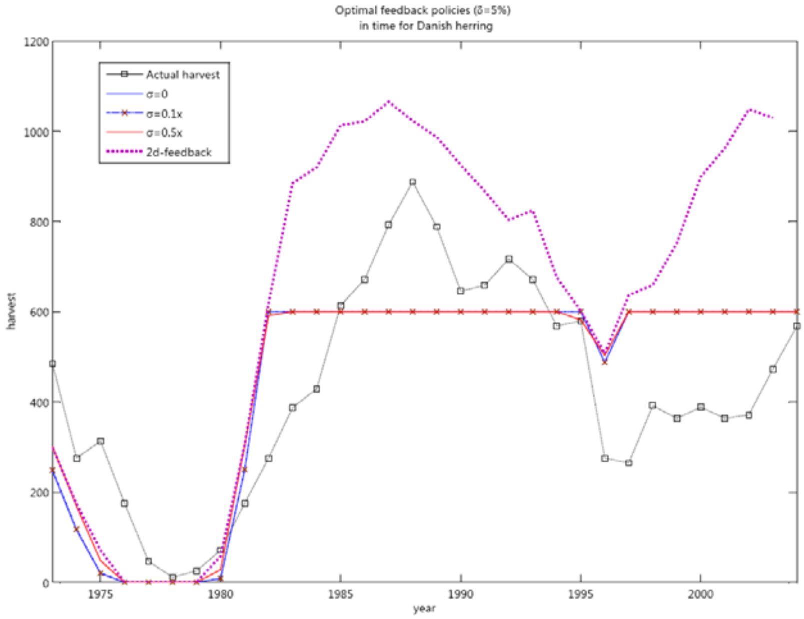


Figure 6.23 Optimal feedback harvest polities for herring (1000 tons).

Taking the interaction of the herring with the cod stock into account leads to the 2d-optimal herring harvesting policy (dashed curve). This represents higher herring catch every year. The reason is that according to our estimates, herring and cod are competing for the same food. Compared with this two-species optimal harvesting policy, the actual herring harvest has been much too low since 1980.

Optimal harvesting policies: Species interactions

Under species interactions, the optimal harvest policy of one species depends on the stock size of the other species. Harvest feed-back diagrams, therefore, need to be three dimensional.

The following two diagrams provide feed-back diagrams for cod and herring, respectively. Figure 6.24 illustrates the optimal feed-back policy for cod. As shown in the figure, there should be no harvesting of cod unless its biomass is excess of 500.000 metric tonne. The size of the herring stock has a very little effect on this and in general the optimal harvest of cod is independent of the level of the herring stock.

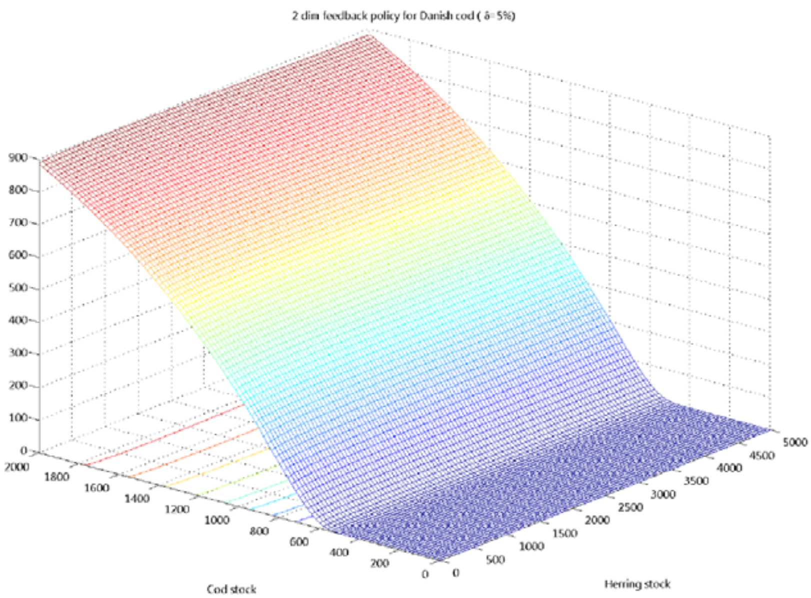


Figure 6.24 Optimal feedback harvest polities for cod with species interaction (1000 tons).

For herring the biomass has to been above 600.000 metric tonne before harvesting is optimal, see Figure 6.25. This level seems to decrease a little with the size of the cod stock. With very high levels of the cod stock the minimum level of the herring stock falls to less than 500.000. Remark, that with very low levels of cod it is optimal to decrease the harvest of herring compared to harvest levels at higher levels of the cod stock. At that point it is optimal to invest in the herring stock.

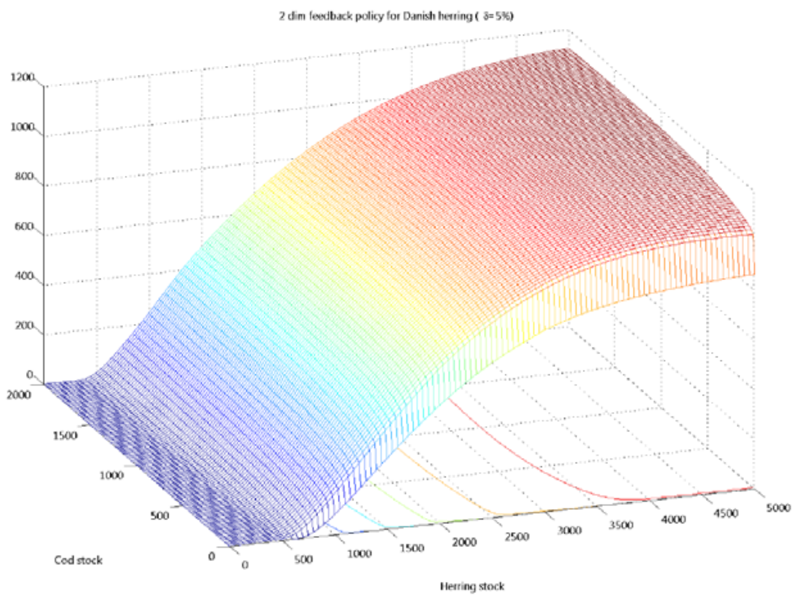


Figure 6.25 Optimal feedback harvest polities for herring with species interaction (1000 tons).

The following phase diagram in biomass space (Figure 6.26) further illustrates the optimal dynamic paths for the biomass of cod and herring from any initial position. Four equilibria exist, but only one of them, located at roughly (cod=850.000 Mt, herring=1.300.000 Mt), is stable. In fact it seems to be globally stable, provided both initial biomasses are positive. At this equilibrium, there will be harvest of both cod and herring, around 200.000 Mt of Cod and 350.000 Mt of Herring. The path to approach this equilibrium is to increase the harvest of herring from the current levels and to close the fishery of cod. When the stock sizes of herring and cod adjust the optimal harvest policy also adjust towards reduced catch levels of herring and at some point positive catch levels of cod.

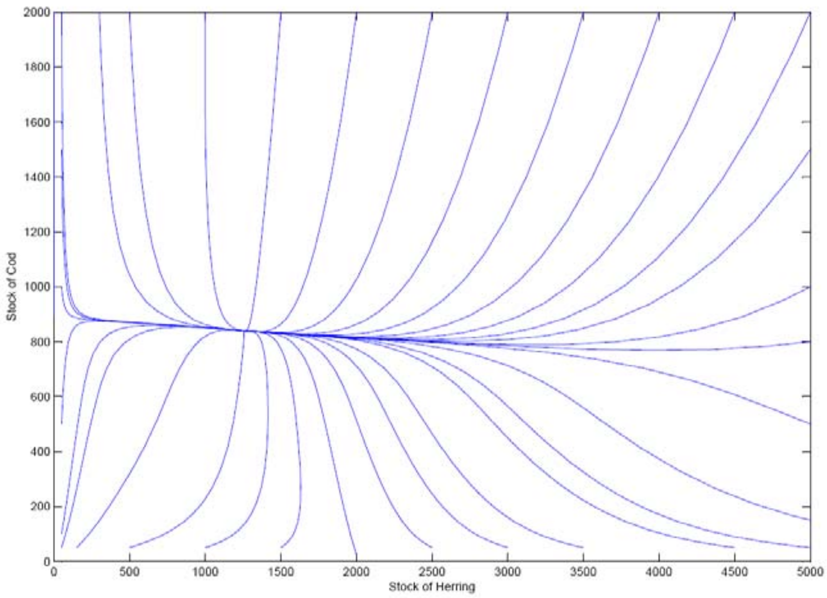


Figure 6.26 Cod-herring biomass: Optimal phase diagram. 1000 tons.

7. Discussion and conclusions

Three different approaches are used to analyze the fisheries harvest policy of cod and capelin/herring in Iceland, Norway and Denmark. The results from the single-species approach - which is an update of earlier work – show that the cod fishery in Iceland and Denmark should be closed and in Norway the harvest should be reduced by 2/3. For capelin/herring, the results are not biased. In the Danish case the harvest of herring could be increased to 600.000 tons. For capelin in Norway the actual harvest fluctuates around the optimal harvest level with tendency towards over harvesting, while for Iceland the actual harvest level is more or less in accordance with the optimal harvest level.

Adding stochasticity to the single species model does not change the results qualitatively. This can be explained by the way uncertainty is handled technical in the model. Current development on uncertainty in fisheries management models shows that uncertainty may arise in different ways and therefore need to be handled more fundamentally. This is an area for future research.

Allowing the species interaction between cod and capelin/herring provides on the other hand new results and insight. In the Danish case the two species model implies a less conservative harvesting pattern for both species. In fact, the current harvest of herring could according to the result be doubled. This is not an obvious result as the harvesting pattern in two species model depends on competitive relationship between the species which are endogenously determined in the model. However, there is a need to explore the biological interaction between cod and herring in more detail. In the case of Iceland the predator-prey model implies more conservative harvesting pattern for both species, particularly the harvest of capelin should - compared to the single-species model and the actual harvest level – be reduced. Both for Denmark and Iceland the difference is significant and uniform over time. In the case of Norway, the predator-prey model implies a more complicated harvesting pattern, and the difference between the single-species and two-species model is not that significant. Furthermore, it is not uniform over time either. On average, however, the two-species model implies a more conservative pattern.

8. References

Arnason, R., L. K. Sandal, S. I. Steinshamn, N. Vestergaard, S. Agnarsson and F. Jensen, 2000. *Comparative Evaluation of the Cod and Herring Fisheries in Denmark, Iceland and Norway*. TemaNord 526.

Arnason, R., Sandal, L.K., Steinshamn, S.I., Vestergaard, N., 2004, Optimal Feedback Controls: Comparative Evaluation of the Cod Fisheries in Denmark, Iceland and Norway, *American Journal of Agricultural Economics*, 86(2): 531–542.

Charles, A.T., 1998, Living with uncertainty in fisheries: analytical models, management priorities and the Canadian ground-fishery experience, *Fisheries Research* 37: 37–50.

Conrad, J.M., 1992, A bioeconomic model of the Pacific whiting, *Bulletin of Mathematical Biology* 54: 219–239.

Gordon, H. Scott. 1954. The Economic Theory of a Common Property Resource: The Fishery. *Journal of the Political Economy* 62(April): 124-42.

Grafton, R.Q., Sandal, L.K. and Steinshamn, S.I., 2000, How to improve the management of renewable resources: The case of Canada's northern cod fishery, *American Journal of Agricultural Economics* 82: 570–580.

Jakobsson, J and G. Stefansson. 1998. Rational harvesting of the cod–capelin–shrimp complex in the Icelandic marine ecosystem. *Fisheries Research* 37:7–21.

Kaitala, V., 1993, Equilibria in a stochastic resource management game under imperfect information, *European Journal of Operational Research* 71: 439–453.

Marine Research Institute. 2006. *State of Marine Stocks in Icelandic Waters 2005/2006. Prospects for the Quota Year 2006/2007*. Marine Resource Institute. Reykjavik.

McDonald, A.D., and Sandal, L.K, 1999, Estimating the parameters of stochastic differential equations using a criterion function based on the Kolmogorov-Smirnov statistic, *Journal of Statistical Computation and Simulation* 64: 235–250.

McDonald, A.D., Sandal, L.K. and Steinshamn, S.I., 2002, Implications of a Nested Stochastic/Deterministic Bio-Economic Model for a Pelagic Fishery", *Ecological Modeling* 149: 193–201.

Milliman, S.R., Johnson, B.L., Bishop, R.C., Boyle, K.J., 1992, The bio-economics of resource rehabilitation – A commercial-sport analysis for a Great-Lakes fishery, *Land Economics* 68: 191–210.

Myers, R.A., and Mertz, G., 1998, Reducing uncertainty in the biological basis of fisheries management by meta-analysis of data from many populations: A synthesis, *Fisheries Research,* 37: 51–60.

Nandram, B., Sedransk, J., Smith, S.J., 1997, Order-restricted Bayesian estimation of the age composition of a population of Atlantic cod, *Journal of the American Statisitical Association* 92: 33–40.

Parsons, L.S. and Lear, W.H., 2001, Climate variability and marine ecosystem impacts: a North Atlantic perspective, *Progress in Oceanography* 49: 167–188.

Rose, G., Gauthier, S., Lawson, G., 2000, Acoustic surveys in the full monte: Simulation uncertainty, *Aquatic Living Resources* 13: 367–372.

Sandal, L. K. and S. I. Steinshamn, 1997a. A Feedback Model for the Optimal Management of Renewable Natural Capital Stocks. *Canadian Journal of Fisheries and Aquatic Sciences* 54: 2475–82.

Sandal, L. K. and S. I. Steinshamn, 1997b. A Stochatic Feedback Model for the Optimal Management of Renewable Resources. *Natural Resource Modeling* 10: 3–52.

Sandal, L.K. and Steinshamn, S.I., 1998, En ikke-lineær modell for optimal ressursforvaltning», *Norsk Økonomisk Tidsskrift* 12: 61-86.

Sandal, L.K. and Steinshamn, S.I., 2001a, A simplified feedback approach to optimal resource management, *Natural Resource Modeling* 14: 419–432.

Sandal, L.K. and Steinshamn, S.I., 2001b, A bioeconomic model for Namibian pilchard, *South-African Journal of Economics* 69: 299–318.

Sandberg, P., Bogstad, B., and Røttingen, I., 1998, Bioeconomic advice on TAC – The state of the art in the Norwegian fishery management, *Fishery Reserarch* 37: 259–274.

Senina, I., Tyutyunov, Y., Arditi, R., 1999, Extinction risk assessment and optimal harvesting of anchovy and sprat in the Azov Sea, *Journal of Applied Ecology* 36: 297–306.

Ulltang, Ø., 1996, Stock assessment and biological knowledge: Can prediction uncertainty be reduced? *ICES Journal of Marine Science* 53: 659–675.

Ussif, A.A., Sandal, L.K. and Steinshamn, S.I., 2002a, On the dynamics of commercial fishing and parameter identification, *Marine Resource Economics*, 2002, 17: 35–46.

Ussif, A.A., Sandal, L.K. and Steinshamn, S.I., 2002b, Estimation of biological and economic parameters of a bioeconomic fisheries model using dynamical data assimilation, *Journal of Bioeconomics*, 4(1): 39–48.

Ussif, A.A., Sandal, L.K. and Steinshamn, S.I., 2003, A new approach of fitting biomass dynamics models to data, *Mathematical Biosciences*, 182(1): 67–79.

Warming, Jens (1911): “Om “Grundrente” af Fiskegrunde”, *Nationaløkonomisk Tidsskrift* 1911,pp. 499-505.

Watson, R.A. and Sumner, N.R., 1999, Uncertainty and risk associated with optimised fishing patterns in a tropical panaeid fishery, *Environment International* 25: 735–744.

Appendix 1 Statistical results for Norway

In the following capelin is denoted by *x*, cod by *y*, harvest of capelin by h_x and harvest of cod by h_y. Everything else are parameters. Stock and harvest are measured in 1000 tons. Revenue and costs are measured in million NOK. Prices are NOK/kg.

Economic model

Demand function capelin:
$p(h_x) = 1$.

Cost function capelin:
$c(h_x) = \alpha \cdot h_x^{1.4}$

parameter	t-value	
α = 0.07.	32.12	R^2 = 0.98
	DW = 1.8	n = 5
β = 1.4.		

The Norwegian share of capelin over the last years has been approximately 60 % on average. Therefore the net revenue function is given by

$$NR(x, h_x) = 0.6 \cdot h_x - c(0.6 \cdot h_x).$$

Demand function cod:
$p(y, h_y) = a + b \cdot h_y$

Parameter	t-value	R^2 = 0.59
	DW = 1.3	n = 11
a = 12.65	9.7	F = 15.6
b = -0.00839	-3.94	

Cost function cod:

$$C(y, h) = k \frac{h^{1.1}}{y}$$

Parameter	t-value	$R^2 = 0.95$
	n = 8	
$k = 5848.1$	44.7	

As this cod is shared 50-50 with Russia, the Norwegian net revenue function is given as

$$.NR(y,h_y) = p(h_y)\cdot\frac{h_y}{2} - C\left(y,\frac{h_y}{2}\right)$$

Biological single species model

Growth function for cod:

$$f(y) = r\cdot y^2\left(1-\frac{y}{K}\right)$$

Parameter	t-value	$R^2 = 0.54$
	DW = 1.6	n = 26
r = 0.000665	12.64	F = 30.83
K = 2 473	25.28	

Growth function for capelin:

$$f(x) = r\cdot x^2\cdot\left(1-\frac{x}{K}\right)$$

Parameter	t-value	$R^2 = 0.62$
	DW = 1.2	n = 27
r = 0.00021781	5.51	F = 44.31
K = 8 293	18.22	

Biological multi-species model

Biological interdependent growth functions:

$$\dot{x} = f(x,y) - h_x$$

$$\dot{y} = g(x,y) - h_y$$

Statistical results (Method: Seemingly Unrelated Regression)

$$f(x,y) = a_1x^2 + b_1x^3 + c_1xy$$

Parameter	t-value	$R^2 = 0.59$
	DW = 1.7	n = 30
$a_1 = 0.00018$	4.9	
$b_1 = -1.19E-8$	-3.1	
$c_1 = -0.00021$	-3.4	

$$g(x, y) = a_2 y^2 + b_2 y^4 + c_2 xy$$

Parameter	t-value	$R^2 = 0.50$
	DW = 1.4	n = 30
$a_2 = 0.00022$	8.4	
$b_2 = -3.49E-11-4.2$		
$c_2 = 1.82E-5$	2.6	

Appendix 2 Statistical results for Iceland

Data

Biomass growth functions for cod and capelin were estimated for the period 1978-2004 with data drawn from ICES (2004) and the Icelandic National Institution of Marine Research (2005), *i.* Hafrannsóknastofnun).

During this period the size of the fishable cod stock (4 years and older) has declined substantially. It peaked at 1200 thousand tons in 1980 but shrank to 400 thousand ton in 1992 before recovering somewhat.

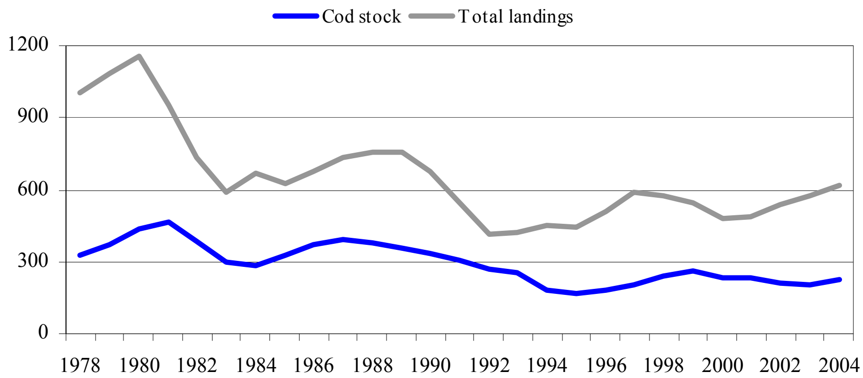


Figure 1. Development of the Icelandic cod stock and total landings 1978-2004. Thousand tons

Reference: Hafrannsóknastofnun.

During the period 1955-1975 Icelandic vessels accounted for about half of the total catch of cod, but that share increased rapidly following the extension of the fishing zone from 12 to 50 miles in 1972 and to 200 miles in 1975. Since then, virtually all of the cod landings have been Icelandic.

The capelin stock (sum of immature and mature capelin in the month of August each year) showed almost uninterrupted decline from 1978 to 1982, finally shrinking to an all time low of 1000 thousand tons at the end of that period. However, the capelin stock recovered quickly and was measured at 3100 thousand tons in 1986. Since then the capelin stock has varied between 1300 and 3000 thousand tons.

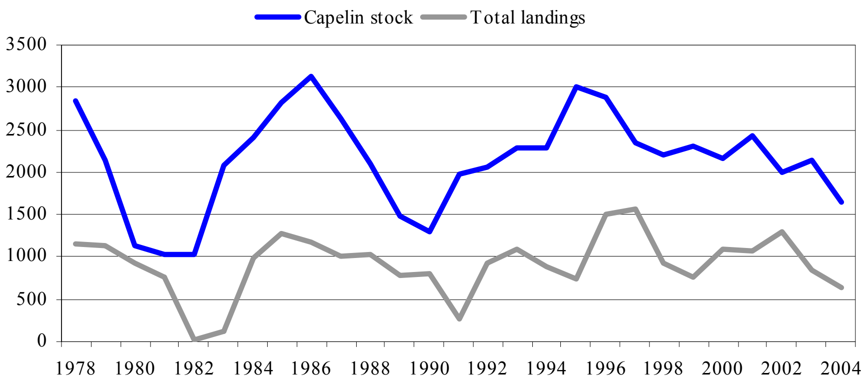


Figure 2. Development of the capelin stock and total landings 1978-2004. Thousand tons

Data for the cost functions are obtained from the National Statistical Institute of Iceland (Statistics Iceland). The data covers the years 1995–2004 and consist of yearly observations on individual vessels in the sample. These data are confidential obtained by special permission to be used only for econometric estimation in this project. The demersal vessel sample is restricted to freezer trawlers. Table 3.1 presents the number of vessels included in the dataset each year. The data includes information on vessels characteristics, costs, sales, annual stock and catch in tons. Cost and sales were deflated using the consumer price index taking a value of unity in 2004 for the simple equations, and converted into Norwegian kronor (NOK). Descriptive statistics for the data are given for demersal species in Table 3.2 and for pelagic species in Table 3.3.

Table 1. Number of vessels observed each year

Year	Demersal fisheries Freezer trawler	Pelagic fisheries Vessel
1995	23	15
1996	19	22
1997	21	23
1998	19	23
1999	20	25
2000	19	29
2001	17	23
2002	18	23
2003	11	21
2004	7	15
Total	174	219

Table 2. Descriptive statistic for vessels engaged in demersal fisheries

	Mean	Std.Dev.	Minimum	Maximum
	Freezer trawlers			
Cost variables				
Variable costs (million NOK)	64.33	49.17	12.75	279.68
Output variables				
Cod harvest (thousand tons)	1.51	1.06	0.02	7.13
Other demersal harvest (thousand tons)	3.18	2.32	0.14	16.65
All demersal harvest (thousand tons)	4.69	2.99	0.61	21.93
Fish stocks				
Cod stock (thousand tons)	694.68	83.28	553.00	854.00
Other demersal stock (thousand tons)	259.56	84.58	197.00	546.00
All demersal stock (thousand tons)	954.24	141.78	780.00	1400.00

Table 3. Descriptive statistic for vessels engaged in pelagic fisheries

	Mean	Std.Dev.	Minimum	Maximum
Cost variables				
Variable costs (million NOK)	23.63	13.85	2.05	74.96
Output variables				
Capelin harvest (thousand tons)	22.02	10.24	0.00	57.64
Herring harvest (thousand tons)	5.96	3.34	0.00	14.40
All pelagic harvest (thousand tons)	33.66	16.91	0.94	93.28
Fish stocks				
Capelin stock (thousand tons)	1737.60	1031.54	0.00	2885.00
Herring stock (thousand tons)	397.28	130.06	0.00	590.00
All pelagic stock (thousand tons)	2134.88	1048.97	0.00	3273.00

Data used for estimation of the inverse demand function is obtained from the National Statistical Institute of Iceland (Statistics Iceland 2006) and consist of monthly observations on landed catches and average prices during the period 2001-2005. Prices are deflated using the consumer price index, and converted into NOK. Catches are expressed in thousand tons and prices in NOK/Kg.

Table 4. Descriptive statistic for Cod

	Mean	Std.Dev.	Minimum	Maximum
		Cod		
Catch (thousand tons)	18,23	2,15	11,25	24,37
Price (NOK/kg)	23,07	3,24	17,47	28,60

Table 5. Descriptive statistic for Capelin

	Mean	Std.Dev.	Minimum	Maximum
	Capelin			
Catch (thousand tons)	114,57	44,89	14,92	278,23
Price (NOK/kg)	1,12	0,37	0,53	2,46

In the following capelin is denoted by x, cod by y, harvest of capelin by h_x and harvest of cod by h_y. Everything else is parameters. Stock and harvest are measured in 1000 tons. Cost is measured in million NOK. Prices are NOK/kg.

Estimation of functions related to the cod fishery

Growth function for cod:

$$f(y) = r \cdot y\left(1 - \frac{y}{K}\right)$$

Parameter	Value	t-statistic	Other properties
r K	0,669853 1988	8,55 -2,93	R^2=0,26 F=8,6

Demand function cod:

$$p(y, h_y) = a - b \cdot h_y$$

Parameter	Value	t-statistic	Other properties
A b	20,96 0,00426	5,46 -2,45	R^2=0,096 F=6,02

Cost function cod:

$$C(y, h) = k\frac{h}{y}$$

Parameter	Value	t-statistic	Other properties
k	5363,179	6,45	R^2=0,43

The parameter 1.1 was found by trying different alternative values and picking the one that yielded the highest R^2.

Estimation of functions related to the capelin fishery

Growth function for capelin:

$$f(x) = r \cdot x \cdot \left(1 - \frac{x}{K}\right)$$

Parameter	Value	t-statistic	Other properties
r K	1,1008 3669	6,325 -3,848	R^2=0,26 F=14,8

Demand function capelin:

$$p(h_x) = a - bh$$

Parameter	Value	t-statistic	Other properties
A B	1,211 0,0001	14,83 -2,58	R^2=0,14 F=5,43

Cost function capelin:

$$c(h_x) = \alpha \cdot h_x^{\,2}$$

Parameter	Value	t-statistic	Other properties
α	0,000175	5,042	R^2=0,309 F=33,35

The exponent ‘2’ was found by trying different alternative values and picking the one that yielded the highest R^2.

Estimation of functions related to the cod fishery

Biological interdependent growth functions:

$$\dot{x} = f(x, y) - h_x$$

$$\dot{y} = g(x, y) - h_y$$

Where

$$f(x, y) = a_1 x + b_1 x^2 + c_1 xy$$
$$g(x, y) = a_2 y + b_2 y^2 + c_2 xy$$

The model was estimated by applying Seemingly Unrelated Regression, SUR, (Zellner 1962, 1963) estimation technique. The data period is from 1978-2004.

Parameter	Value	t-statistic	Other properties
A1	1,4734	5,6834	R^2=0,40
B2	-0,0004	-4,6187	
C1	-0,0004	-1,8102	
A2	0,3518	2,9267	R^2=0,42
B2	-0,0002	-2,1237	
C2	0,0001	3,1298	

References

ICES (2004). *Report of the Northern Pelagic and Blue Whiting Fisheries Working Group*. http://www.ices.dk/reports/ACFM/2004/WGNPBW/directory.asp

Nytjastofnar sjávar 2004/2005: Aflahorfur fiskveiðiárið 2005/2006 (2005). Reykjavík: Marine Research Institute.

Statistics Iceland (2006). Web site: http://www.statice.is/.

Zellner, Arnold, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests of Aggregation Bias," JASA 57 (1962), pp. 348-368.

Zellner, Arnold, "Estimators for Seemingly Unrelated Regression Equations: Some Exact Finite Sample Results," JASA 58 (1963), pp. 977-992.

Appendix 3 Statistical results for Denmark

Growth function cod

Data

Data for for cod in North Sea comes from ICES Advisory Committee on Fishery Management (2004, Table 3.4.9 Cod in Subarea IV and Divisions IIIa (Skagerrak) and VIId: Stock summary as estimated by ADAPT without discards), it is in 1.000 ton. Growth at time t for is calculated as

$$g_t = X_{t+1} - X_t + h_t$$

Where X is biomass and h is harvest. The data set is given in table 1.

Table 1. Biomass and growth for North Sea cod in 1.000 ton

	Year	Biomass	Growth
1	1963	448.184	194.904
2	1964	526.631	280.101
3	1965	680.691	326.380
4	1966	826.035	289.811
5	1967	894.510	117.325
6	1968	758.858	134.691
7	1969	605.181	522.408
8	1970	926.829	432.571
9	1971	1133.276	-10.658
10	1972	794.520	190.559
11	1973	631.103	213.229
12	1974	605.281	288.824
13	1975	679.826	109.775
14	1976	584.356	444.801
15	1977	794.988	189.503
16	1978	775.337	290.855
17	1979	769.170	476.345
18	1980	975.542	138.436
19	1981	820.334	321.544
20	1982	806.381	118.468
21	1983	621.598	329.604
22	1984	691.915	19.863
23	1985	483.492	391.936
24	1986	660.799	98.747
25	1987	555.493	72.530
26	1988	411.811	178.747
27	1989	406.318	57.372
28	1990	323.754	104.047
29	1991	302.487	242.958
30	1992	442.967	85.072
31	1993	414.019	301.812
32	1994	594.082	105.932
33	1995	589.380	33.831
34	1996	487.115	212.138
35	1997	572.933	-92.514
36	1998	356.261	91.272
37	1999	301.519	58.701
38	2000	263.995	15.515
39	2001	208.139	82.923
40	2002	241.430	-2.361
41	2003	184.204	NA

Model

There is assumed a logistic growth function, that is, the model is:

$E(g_t)$ ˜ ! X_t —" X_t^2

(1)

An ordinary least square estimate gives the statistics given in table 2.

Table 2. Estimates and statistics from an ordinary least square estimate of the model (1)

Parameter	Estimate	Std.error	t-value	p-value
□	0.6028	0.1331	4.527	5.74e-05
□	-0.0004206	0.0001738	-2.42	0.02041

Residual standard error: 139 on 38 degrees of freedom

Multiple R-Squared: 0.1203 Adjusted R-squared: 0.0972

F-statistic: 5.1975 on 1 and 38 DF, p-value: 0.02832

The R and F statistics compares the residuals of the model with the residuals of the model *E(g)* = α. Note however; as the later is not a submodel of the former the general logic of variance analysis do not apply.

Durbin-Watson

lag 1	lag 2	lag 3	lag 4
2.194049	1.663057	1.628194	1.800477.

Both parameters are significant in the t-statistics and there seems to be no autocorrelation. The model is accepted for final model. In figure 1 the observations and the model predictions is plotted.

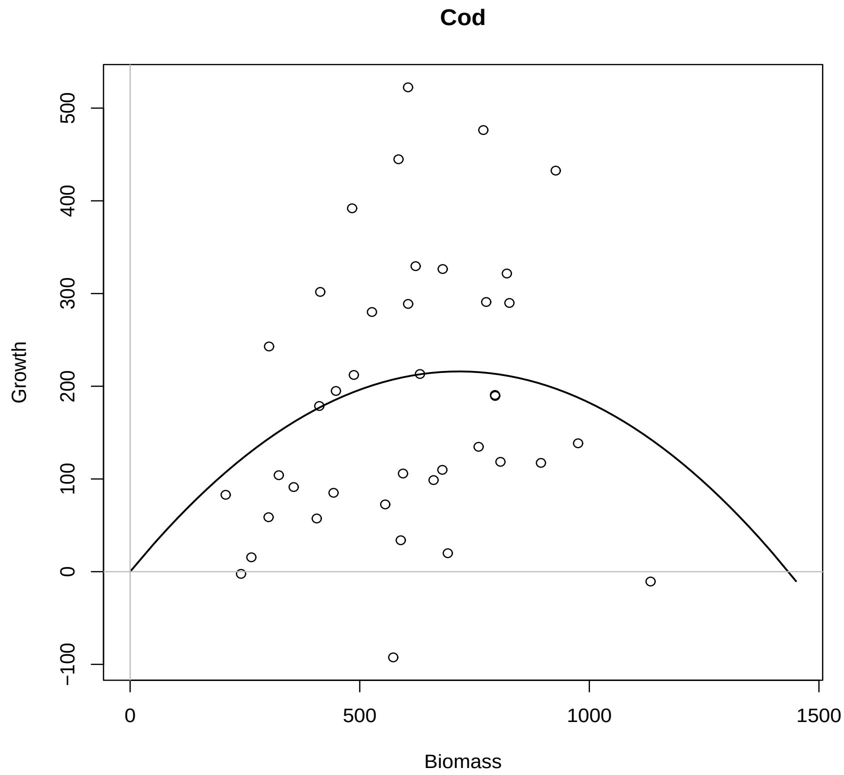


Figure 1. Observations and model predictions for growth of cod in the North Sea

Conclusion

The growth of cod can be modeled as:

$$E(g_t) = \alpha X_t - \beta X_t^2$$

with the parameter given in table 2.

If the model is written as

$$E(g_t) = rX_t\left(1 - \frac{X_t}{K}\right)$$

the parameters are

Parameter	Estimate	
r	0.6028	year^{-1}
K	1433	10^3ton

Demand function cod

Data

Data from Arnason *et al*. (2004) is updated with Fiskeridirektoratet (2000, 2001, 2002, 2003, 2004, tabel 3.1) so the time series is now 1982-2004, i.e. 23 observations. Harvest in ton and value in 1.000 DKK.

Price is calculate as value divided by landings, hence price is in 1.000 DKK pr. ton or DKK pr. kg. Nominal price is converted to real price with CPI (Danmarks Statistik, 2006) with base of 2004 and converted to NOK by exchange rate 100DKK=90.9300NOK (1/6 2004). The data set is given in table 3.

Table 3. Landings in ton and real price (2004) in NOK for Denmark

	Year	Landings	Realprice
1	1982	160440	13.11377
2	1983	155567	12.97773
3	1984	161296	12.94424
4	1985	144701	13.72785
5	1986	129352	15.92733
6	1987	127685	15.28808
7	1988	108070	14.24944
8	1989	99111	14.55533
9	1990	86373	17.97972
10	1991	74842	19.15734
11	1992	55459	18.49239
12	1993	40863	15.56179
13	1994	47882	14.80385
14	1995	67456	12.50697
15	1996	78097	11.26131
16	1997	69184	13.25102
17	1998	57937	17.38752
18	1999	59822	18.03741
19	2000	48256	19.43928
20	2001	39724	20.32533
21	2002	32616	20.61981
22	2003	26988	17.23244
23	2004	26346	16.62473

Model

A linear model is used to model the real price:

$$p_i = \alpha + \beta h_i + \varepsilon_i$$

p_i average real price in NOK pr.kg. (or 1.000 NOK pr ton) of cod in Denmark in year i, h_i is the amount of cod in ton landed in Denmark in year i and $i = 1982, 1983, \ldots, 2004$. This model yields residuals with high autocorrelation, therefore the model is attempted corrected with autocorrelation of the AR(1), AR(2) and AR(3) type:

model I	%ₜ	assumed	NID(0,3 ^{2})
model II	% ˜ / $_1$ %ₜ™₁ —, $_i$	where ν_i assumed	NID(0,3 ^{2})
model III	% ˜ / $_1$ %ₜ™₁ —/ $_2$ %ₜ™₂ —, $_i$	where ν_i assumed	NID(0,3 ^{2})
model IV	% ˜ / $_1$ %ₜ™₁ —/ $_2$ %ₜ™₂ —/ $_3$ %ₜ™₃ —, $_i$	where ν_i assumed	NID(0,3 ^{2})

The models estimated with generalized least squares fitted by maximum likelihood (`gls( ,method="ML")` Pinheiro et al., 2006) gives the statistics as given in table 4, and in figure 2 the four models are plotted together with the data.

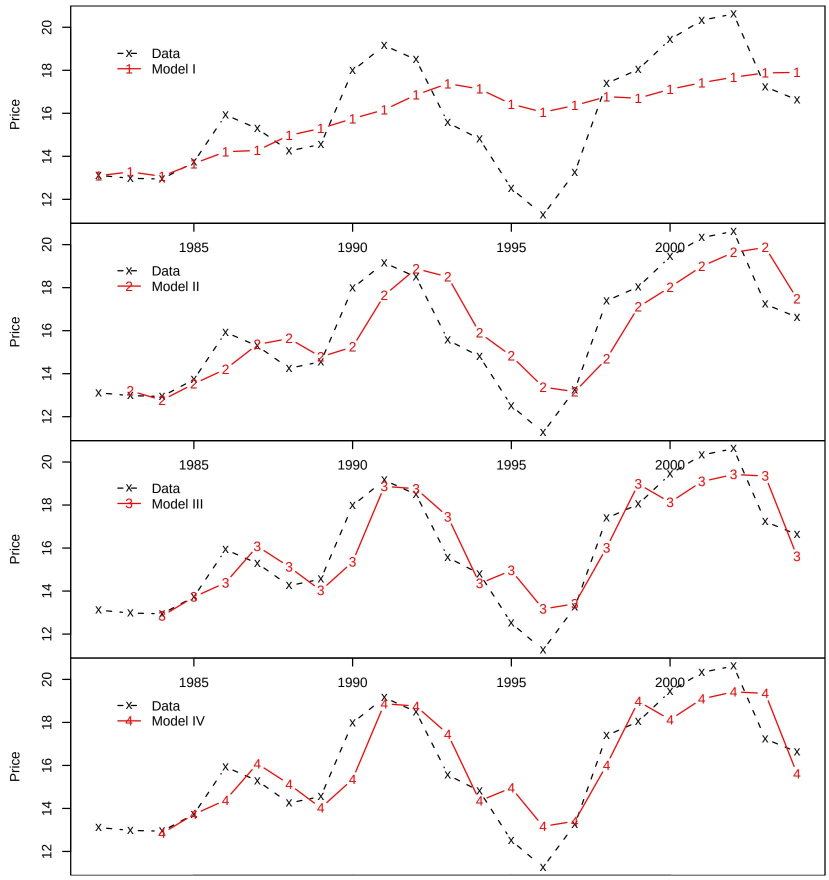


Figure 2. The four models prediction including the autocorrelation part plotted together with the data

Table 4. Statistics for generalized least squared estimates

	Par	LogLik	Sigma	Lag 1	Lag 2	Lag 3	Lag 4
Model I	2	-50.1162	2.1383	0.6177	1.6319	2.2788	2.5992
Model II	3	-43.0139	1.5843	1.1840	2.0744	2.3618	2.0981
Model III	4	-38.9312	1.3365	1.9590	1.9923	2.0252	1.1907
Model IV	5	-38.8564	1.3643	1.9252	2.0850	1.9546	1.1315

Par refers to numbers of parameters and "Lag *n*" relates to the Durbin-Watson statistic of the residual with lag *n*.

The Durbin-Watson is acceptable for model III and the improvement in the likelihood from model III to model IV is very small, therefore model III is accepted as final model. In table 5 is given parameter estimates for model III. Contrary to previous (Arnason *et al.*, 2004) the β is now significant and price is now correlated with harvest.

Table 5. Parameter estimates and statistics for model III

Parameter	Estimate	Std.error	t-value	p-value
ϕ_1	1.043			
ϕ_1	-0.5292			
α	18.66	1.228	15.19	8.386e-13
β	-3.368e-05	1.312e-05	-2.567	0.01795

Multiple R-Squared: 0.7385 Adjusted R-squared: 0.7247

F-statistic: 53.644 on 1 and 19 DF, p-value: 6.059e-07

Deviance: 32.1862 on 3 DF

The F and R statistics compare the residuals of the model with the residuals of a fix price model, $E(p)=\gamma$. As residuals is in model III used v_t. As there in this model are no residuals for the first two observations, the first two observations are left out in the estimation of the fixed price model. Note however; as there in the estimation of model III, as object not is used minimum of the sums of squares, but maximum of likelihood, the general logic of variance analysis do not apply. However, the deviance statistics – minus 2 times the difference in loglikelihood – is asymptotic χ^2_{DF} distributed.

If the autocorrelation part is ignored, the price can be estimated as

$$E(p_t) = \alpha - \beta h_t$$

with the parameters given in table 5. However the landing is referring to the landings in Denmark, not in the North Sea. Landings of cod in Denmark is 0.1883684 of total catch in the North Sea (std.err. 0.061), it is therefore reasonable to anticipate only this fraction of the North Sea harvest will appear on the Danish marked and influence the price. The formula therefore has to be corrected

$$
\begin{aligned}
E(p_t) &= \alpha + \beta h_t \\
&= \alpha + 0.1836\beta H_t \\
&= \alpha + B H_t
\end{aligned}
$$

where H is the total harvest in the North Sea and the $B = -0.006344$ when H is measured in 1000 ton.

Cost function cod

Theory

Total cost for cod harvesting is expected to be of the form

$$C(H,X) = \alpha \frac{H^{\beta}}{X} \qquad (1)$$

Where α and β is parameters and H is total harvest of cod and X is biomass of the cod. If the production is divided into to sectors the total cost can be written as

$$C = \alpha_i \frac{h_i^{\beta}}{X} + \alpha_j \frac{\left(H - h_i\right)^{\beta}}{X}$$

if the cost function is assumed equal for the two sectors i.e. $\alpha_i = \alpha_j$ we have

$$C = \frac{\alpha_i}{X}\left(h_i^{\beta} + \left(H - h_i\right)^{\beta}\right) \qquad (2)$$

Equitation (1) and (2) yields

$$\alpha_i = \frac{\alpha H^{\beta}}{h_i^{\beta} + \left(H - h_i\right)^{\beta}}$$

α and β can therefore be estimated from a single sector empirical cost:

$$
\begin{aligned}
C_i(h_i,X) &= \alpha_i \frac{h_i^{\beta}}{X} \\
&= \alpha \frac{h_i^{\beta} H^{\beta}}{\left(h_i^{\beta} + \left(H - h_i\right)^{\beta}\right)X}
\end{aligned}
\qquad (3)
$$

The accounting statistic for fishery in Denmark has as its basic unit a firm, normally consisting of one fishing vessel. The Danish fishing vessels catch a mixture of fish and operate in both the Baltic and the North See. The fishery in the North Sea is practiced by a lot of nations. As the only segment of the Danish fleet which have the North Sea as there main operation area is the Danish-seine fleet, our approach is to use data for the cost for the Danish fleet and to estimate the total cost in the North Sea with the equation (3). Therefore following the model is used

$$E(C_t) = \alpha \frac{h_t^{\beta} H_t^{\beta}}{\left(h_t^{\beta} + \left(H_t - h_t\right)^{\beta}\right) X_t} \varepsilon \quad (4)$$

Where $E(C_t)$ is the expected variable cost, h_t is the harvest for the Danish-seine fleet in year t, and H_t and X_t are the total harvest and biomass of cod in the North Sea. The parameter α and β can then be used in equation (1) to extrapolate to total costs.

Data

The fishery account statistic from 1995-1998 (Statens Jordbrugs- og Fiskeriøkonomiske Institut, 1997*a,b*, 1998, 1999, 2001; Fødevareøkonomisk Institut, 2005) has data for variable cost, gross output distributed according to species and an estimate of the fisherman's remuneration. From 2000-2004 the account statistic data is stratified on size of vessels. As the Danish-seine vessels is landing a variety of species, the variable cost for cod is calculated so the cods share of cost equal cods share of gross output. In the table 6 data is given for cods share of variable cost and cods share of gross output, all in 1,000DKK for the fleet in total.

Table 6. The share of variable cost and gross output in the Danish danish-seine fleet that is related to cod, all in 1.000DKK

	Year	Gross output	Variabel cost
1	1995	81592.90	76351.66
2	1996	74050.80	63235.70
3	1997	62887.00	50756.57
4	1998	116719.80	94568.97
5	1999	172725.00	136780.91
6	2000	79383.70	71965.72
7	2001	67005.68	56067.97
8	2002	66340.79	59965.06
9	2003	36155.55	33185.60
10	2004	29558.82	29259.84

To calculate the harvest of cod in weight the output of cod is divided by the nominal price for cod (in 1.000DKK pr ton) for that year. The variable cost in nominal prices is converted real price with CPI (Danmarks Statistik, 2006) with 2004 as base, and converted to NOK by exchange rate 100DKK=90.9300NOK (1/6 2004). For total harvest and total biomass of cod in North Sea ,Table 3.4.9 Cod in Subarea IV and Divisions IIIa (Skagerrak) and VIId: Stock summary as estimated by ADAPT without discards is used. In table 7 the final data set is given.

Table 7. Landings in ton and variable cost in 1.000 NOK real price (2004) for the Danish seine fleet and the total harvest and stock biomass of cod in the North Sea in ton

	Year	Landings	Variable cost	Harvest	Stock
1	1995	8707.112	101904.23	136096	589380
2	1996	8594.782	82652.59	126320	487115
3	1997	6069.862	64917.15	124158	572933
4	1998	8430.505	118766.88	146014	356261
5	1999	11734.007	167606.50	96225	301519
6	2000	4862.054	85682.93	71371	263995
7	2001	3834.511	65215.50	49632	208139
8	2002	3653.902	68101.90	54865	241430
9	2003	2334.067	36917.71	30872	184204
10	2004	1955.354	32178.42	NA	NA

Results

Table 8.Estimates and statistics from a nonlinear least square estimate of the model (4)

	Estimate	Std. Error	t value	Pr(> | t |)
alpha	412599e+06	2.869280e+06	0.8408377	4.282231e-01
beta	069016e+00	1.284620e-01	8.3216505	7.080664e-05

Residual standard error: 18398 on 7 degrees of freedom Multiple R-Squared: 0.7952
Adjusted R-squared: 0.7659 F-statistic: 27.1768 on 1 and 7 DF, p-value: 0.001235
Loglikelihood -100.02

The R and F statistics compares the residuals of the model with the residuals of the model $E(C)=\gamma$. Note however; as the later is not a submodel of the former the general logic of variance analysis do not apply.

A nonlinear least square estimate of the model (4) gives the result in table 8. Note that the t-test in the summary is a test with H_0: $\beta = 0$, where as the interesting hypothesis might be $\beta = 1$ or $\beta = 1.1$ – the Norwegian case: Both hypotheses can not be rejected, and if there is special arguments for the Norwegian $\beta = 1.1$ it will be all right with the data. The α and β is highly (negative) correlated, therefore only one is significant. If β is exogenous the α is significant in an ordinary least square estimate. The resulting α estimates together with σ and the log likelihood is given in table 9:

Table 9. Statistics from an ordinary least square estimate with exogenous β

	Sigma	loglik	Estimate	Std. Error	t value	Pr(> | t |)
beta=1	17608.87	-100.23	4561440.62	286238.99	15.936	2.408e-07
beta=1.096	17209.74	-100.02	2412961.41	147881.48	16.317	2.004e-07
beta=1.1	17287.86	-100.06	1811373.28	111531.54	16.241	2.078e-07

The likelihood is natural biggest with $\beta = 1.069$, however the difference in the log likelihood is small, and the $\beta = 1.1$ can be chosen with a theoretical argument. Notice that the t-test is for H_0: $\alpha = 0$, a more relevant test is to test if the cost is fixed, i.e. H_0: $E(C_t) = \gamma$ or if relative cost is fixed, i.e. H_0: $E(C_t) = \gamma h_t$. The number of parameters in the test models and in equation (4) with β as exogenous is the same (i.e. 1) so sigma and log likelihood can be compared see table 10.

Table 10. The residual standard error and the log likelihood statistics from estimation of the models for fixed cost: $E(C_t) = \gamma$ and fixed relative cost $E(C_t) = \gamma h_t$

	sigma	loglik
Fixed cost	39959	-119.62
Fixed relative cost	17370	-111.29

The models in table 9 have all better likelihoods and are therefore preferred. The models in table 10 might as well be compared with the full model where both β and α is estimated, here there is a reduction in parameters from 2 to 1. The models in table 10 are not submodels of the full model, however the likelihood is decreasing so it is safe to reject the fixed cost and fixed relative cost models. As the $\beta = 1.069$ yields the highest likelihood it is chosen for the final model.

The R and F statistics compare the residuals of the model with the residuals of the fix pries model $E(g)=\gamma$. Note however; as the later is not a submodel of the former the general logic of variance analysis do not apply. The residuals from the model with exogenous β have the same residuals as the model estimated with non linear least squared, therefore the R-squared is the same as well: R-Squared equals 0.7952. As the model has only one parameter when β is exogenous, there is no adjustment to bee done; so adjusted R-squared equals 0.7952 as well. As the F statistic is only defined for a compare of two models with different number of parameters, it is not possibly to give any F statistic.

Conclusion

The expected variable cost in the North Sea cod fishery in 1,000 NOK real price (2004) can be estimated by:

$$E(C)^{\sim} \; ! \; \frac{H^{1.069}}{X}$$

Where $\alpha = 2{,}412{,}961$, H is total harvest of cod in ton and X is the North Sea cod biomass in ton.
If the stock and harvest is measured in 1,000 ton and cost in million NOK the formula is the same just with $\alpha' = 1{,}000^{-0.931}\alpha = 3{,}886.426$.

Growth function Herring

Data

Data for herring in North Sea is in 1.000 ton and comes from ICES Advisory Committee On Fishery Management (2005, Table 2.6.2.3 North Sea herring. STOCK SUMMARY). Growth at time t for is calculated as

$$g_t = X_{t+1} - X_t + h_t$$

Where X is biomass and h is harvest. The data set is given in table 11.

Table 11. Biomass and growth for North Sea herring in 1.000 ton

	Year	Biomass	Growth
1	1960	3719.372	1314.803
2	1961	4337.975	739.378
3	1962	4380.653	855.792
4	1963	4608.645	888.691
5	1964	4781.336	419.745
6	1965	4329.881	147.157
7	1966	3308.238	403.083
8	1967	2815.821	400.110
9	1968	2520.431	102.463
10	1969	1905.094	563.507
11	1970	1921.901	490.625
12	1971	1849.426	220.143
13	1972	1549.469	103.996
14	1973	1155.965	239.922
15	1974	911.887	43.349
16	1975	680.136	-8.999
17	1976	358.337	26.608
18	1977	210.145	60.423
19	1978	224.568	168.164
20	1979	381.732	273.481
21	1980	630.113	598.986
22	1981	1158.335	859.395
23	1982	1842.851	1150.531
24	1983	2718.303	532.676
25	1984	2863.777	1025.805
26	1985	3460.951	623.627
27	1986	3470.798	1134.475
28	1987	3933.785	433.882
29	1988	3575.609	617.481
30	1989	3305.404	453.452
31	1990	2970.957	382.931
32	1991	2708.659	380.208
33	1992	2430.859	798.845
34	1993	2512.905	171.843
35	1994	2013.351	368.882
36	1995	1813.999	360.150
37	1996	1594.778	586.701
38	1997	1906.381	357.721
39	1998	1999.789	679.636
40	1999	2287.797	939.325
41	2000	2863.959	759.383
42	2001	3235.185	1168.563
43	2002	4040.405	186.258
44	2003	3855.722	144.795
45	2004	3527.930	NA

Model

There is assumed a logistic growth function, i.e. the model is:

$$E(g_t)\ \tilde{}\ \ !\ X_t — "\ X_t^2 \quad (1)\ (1)$$

An ordinary least square estimate gives the statistics given in table 12

Table 12. The estimates and statistics form an ordinary least square estimate of the model (1)

Parameter	Estimate	Std.error	t-value	p-value
□	0.3715	0.0671	5.537	1.834e-06
□	-5.446e-05	1.864e-05	-2.921	0.005597

Residual standard error: 316 on 42 degrees of freedom
Multiple R-Squared: 0.1903 Adjusted R-squared: 0.171
F-statistic: 9.8696 on 1 and 42 DF, p-value: 0.003076
Loglikelihood -327.86

The R and F statistics compares the residuals of the model with the residuals of the model *E(g)* = α. Note however; as the later is not a submodel of the former the general logic of variance analysis do not apply.

Durbin-Watson

Lag 1	lag 2	lag 3	lag 4
0.8670043	0.9045002	1.1419535	1.2926933

As seen in table 12 there seem to be autocorrelation. This autocorrelation is not, as in prices, caused by adaptive agents. This correlation is caused by repeated measurement on the same observation unit. The residuals can therefore not be expected to be independent distributed. Consequently the correlation between the residuals is modeled with various types of functions.

In table 13 is given statistics for the estimation with 10 different correlation structures. The `mXxx` forms have one parameter extra (without a nugget parameter) and the `nXxx` have two parameter extra (with a nugget parameter). `xExp` means a exponential spatial correlation, `xGaus` means a Gaussian spatial correlation, `xLin` means a linear spatial correlation, `xRatio` means a Rational quadratics spatial correlation, `xSpher` means a spherical spatial correlation (for details on the correlation structures see Pinheiro *et al*. 2006).

Table 13. Statistics for estimation of the model with different correlation structures

	Model	df	AIC	BIC	logLik
m0	1	3	661.7108	666.9238	-327.8554
mGaus	2	4	659.6281	666.5788	-325.8141
mSpher	3	4	655.1810	662.1317	-323.5905
mRatio	4	4	657.0121	663.9628	-324.5060
mLin	5	4	653.4198	660.3704	-322.7099
mExp	6	4	655.1354	662.0861	-323.5677
nGaus	7	5	652.0810	660.7694	-321.0405
nSpher	8	5	655.2902	663.9786	-322.6451
nRatio	9	5	652.8150	661.5033	-321.4075
nLin	10	5	652.0410	660.7293	-321.0205
nExp	11	5	654.2899	662.9783	-322.1450

For explanation of the different correlation structures see the text. The column marked df gives the number of parameters including those in the variation structure.

As seen in table 13 the functional form that yields the best results with both one and two parameters is the `xLin` (linear) type. A compare between the model without correlation and the model with the two linear correlations are given in table 14.

Table 14. Statistics for the estimation of the model with no correlation structure and with a linear correlation structure, with out and with nugget

	Model	df	AIC	BIC	logLik	Test	L.Ratio	p-value
m0	1	3	661.7108	666.9238	-327.8554			
mLin	2	4	653.4198	660.3704	-322.7099	1 vs 2	10.291051	0.0013
nLin	3	5	652.0410	660.7293	-321.0205	2 vs 3	3.378783	0.0660

The column marked df gives the number of parameters including those in the variation structure. The test statistics is the quotient test for the loglikelighoods.

Table 15. Compare of the estimated coefficients in the 11 models

	alpha	beta
m0	0.3715427	-5.445625e-05
mGaus	0.4045798	-6.456684e-05
mSpher	0.6886205	-1.674036e-04
mRatio	0.4421379	-7.604844e-05
mLin	0.4330222	-1.311132e-04
mExp	0.6050297	-1.275548e-04
nGaus	0.5318474	-1.079538e-04
nSpher	0.5239859	-1.098982e-04
nRatio	0.5348822	-1.102015e-04
nLin	0.5442206	-1.111579e-04
nExp	0.5460848	-1.087074e-04

It could, based upon the quotient test in table 14, be conclude that the one without the nugget (`mLinn`) is satisfactorily, but if the coefficients estimates for the 11 models is compared, see table 15, it is obviously that the one parameter versions (`mXxx`, without nugget) give quit different parameter estimates, while the two parameter version (`nXxx`, with nugget) yields stable parameter estimates. The version with linear correlations matrix with the nugget is therefore accepted as final model:

$$g_t = \alpha X_t + \beta X_t^2 + \varepsilon_t$$

$$\mathrm{cor}(\varepsilon_i, \varepsilon_j) = \begin{cases} (1-n)\left(1-\dfrac{|i-j|}{d}\right) & if\ |i-j| < d \\ 0 & if\ |i-j| \geq d \end{cases}$$

where *d* is the range and *n* is the nugget parameter. Parameter estimates is given in table 16 and in figure 3 a plot of observations and model is given.

Table 16. Parameter estimates for model (2)

Parameter	Estimate	Std.error	t-value	p-value
□	0.5442	0.128	4.252	0.0001156
□	-0.0001112	3.035e-05	-3.663	0.0006924
D	8.163			
N	0.2021			

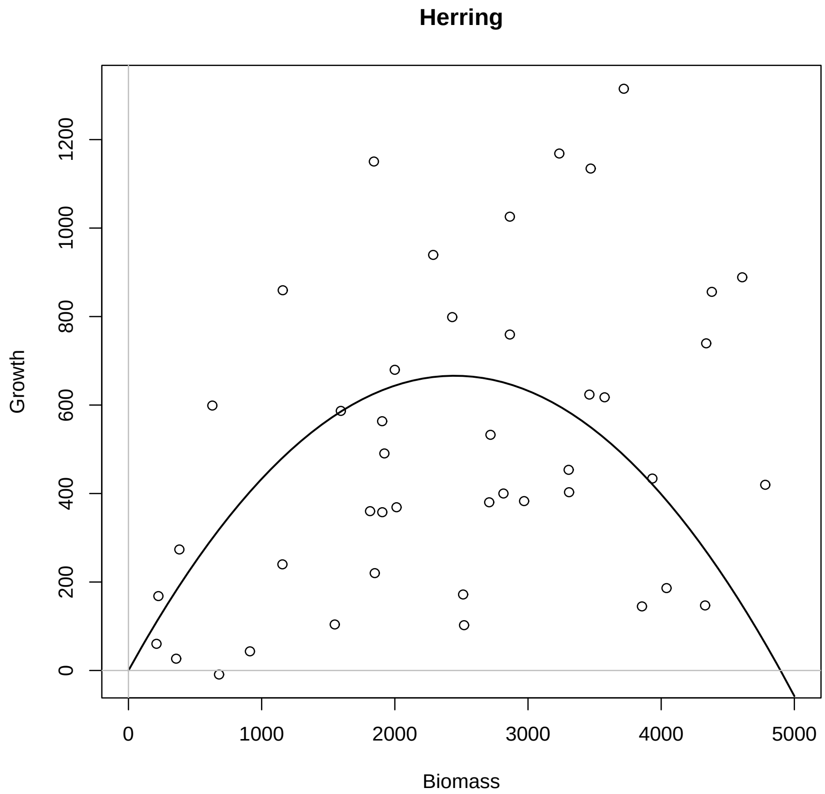


Figure 3. Observations and model predictions for growth of herring in the North Sea

Conclusion

The growth of herring can be modeled as:

$$E(g_t) = \alpha X_t - \beta X_t^2$$

with the parameter given in table 16.

If the model is written as

$$E(g_t) = rX_t\left(1 - \frac{X_t}{K}\right)$$

the parameters are

Parameter	Estimate	
r	0.5442	year^{-1}
K	4896	10^3ton

The model suggests that the residuals are correlated, because of this correlation there might be an expected value for the residual next year different form zero, in other words: the model can not be expected to give unbiased predictions.

Demand function Herring

Data

Data from Arnason *et al.* (2000) is updated with Fiskeridirektoratet (2006*a,b*), so the time series is now 1982-2005, i.e. 24 observations. Harvest in 1.000 ton and value in 1.000 DKK. Price is calculate as value divided by landings, and given as price is in 1.000 DKK pr. ton or DKK pr. kg. Nominal price is converted to real price with CPI (Danmarks Statistik, 2006) with base of 2004 and converted to NOK by exchange rate 100DKK=90.9300NOK (1/6 2004). The data set is given in table 17.

Table 17 Landings in 1.000 ton and real price (2004) in NOK for Denmark

	Year	landing	price
1	1982	81.000	3.864613
2	1983	172.000	2.204645
3	1984	124.000	2.778501
4	1985	136.000	2.969786
5	1986	150.000	2.248954
6	1987	157.000	1.816671
7	1988	184.000	1.835043
8	1989	171.000	2.018763
9	1990	136.000	2.183031
10	1991	146.000	2.140883
11	1992	156.000	2.175466
12	1993	169.000	1.918257
13	1994	178.000	1.853415
14	1995	191.000	1.463279
15	1996	153.009	1.356541
16	1997	125.302	1.524046
17	1998	139.711	1.558200
18	1999	137.578	1.298876
19	2000	153.899	1.129468
20	2001	141.508	2.220865
21	2002	112.582	2.457409
22	2003	114.806	1.729254
23	2004	136.809	1.521349
24	2005	167.450	1.881092

Model

A linear model is used to model the real price:

$$p_i = \alpha + \beta h_i + \varepsilon_i$$

where p_i is average real price in NOK pr.kg. (or 1.000 NOK pr ton) of herring in Denmark in year i, h_i is the amount of herring in ton landed from Danish fishing vessels in year i and $i = 1982, 1983, \ldots, 2005$. This model yields residuals with high autocorrelation, hence the model is attempted corrected with autocorrelation of the AR(1), AR(2). This do however not yield god results and moving average is included in the modeling in the form of the ARMA(0,1), ARMA(1,1), ARMA(0,2), ARMA(1,2) and ARMA(0,3) type:

model (0,0)	ε_i	assumed	$\mathrm{NID}(0,\sigma^2)$
model (1,0)	$\varepsilon_i = \phi\varepsilon_{i-1} + \nu_i$	where ν_i assumed	$\mathrm{NID}(0,\sigma^2)$
model (2,0)	$\varepsilon_i = \phi_1\varepsilon_{i-1} + \phi_2\varepsilon_{i-2} + \nu_i$	where ν_i assumed	$\mathrm{NID}(0,\sigma^2)$
model (0,1)	$\varepsilon_i = \theta\nu_{i-1} + \nu_i$	where ν_i assumed	$\mathrm{NID}(0,\sigma^2)$
model (1,1)	$\varepsilon_i = \phi\varepsilon_{i-1} + \theta\nu_{i-1} + \nu_i$	where ν_i assumed	$\mathrm{NID}(0,\sigma^2)$
model (0,2)	$\varepsilon_i = \theta_1\nu_{i-1} + \theta_2\nu_{i-2} + \nu_i$	where ν_i assumed	$\mathrm{NID}(0,\sigma^2)$
model (1,2)	$\varepsilon_i = \phi\varepsilon_{i-1} + \theta_1\nu_{i-1} + \theta_2\nu_{i-2} + \nu_i$	where ν_i assumed	$\mathrm{NID}(0,\sigma^2)$
model (0,3)	$\varepsilon_i = \theta_1\nu_{i-1} + \theta_2\nu_{i-2} + \theta_3\nu_{i-3} + \nu_i$	where ν_i assumed	$\mathrm{NID}(0,\sigma^2)$

The models estimated with generalized least squares fitted by maximum likelihood (`gls( ,method="ML")` Pinheiro et al., 2006) gives the statistics as given in table 18, and in figure 4 the eight models are plotted together with the data.

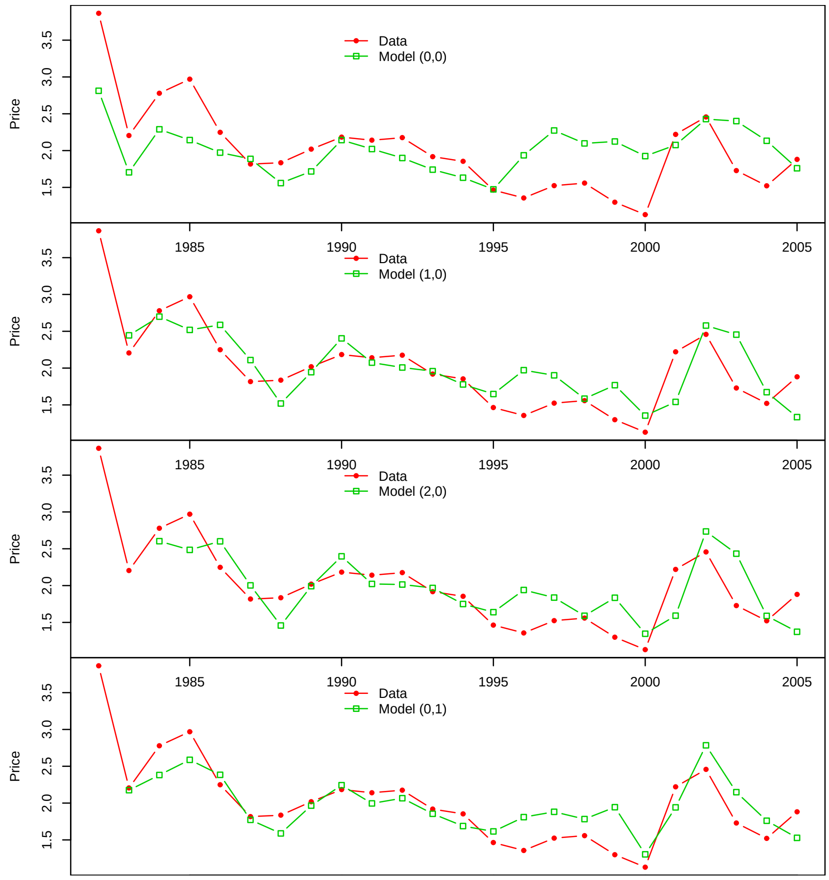


Figure 4. The models prediction including the autocorrelation part plotted together with the data

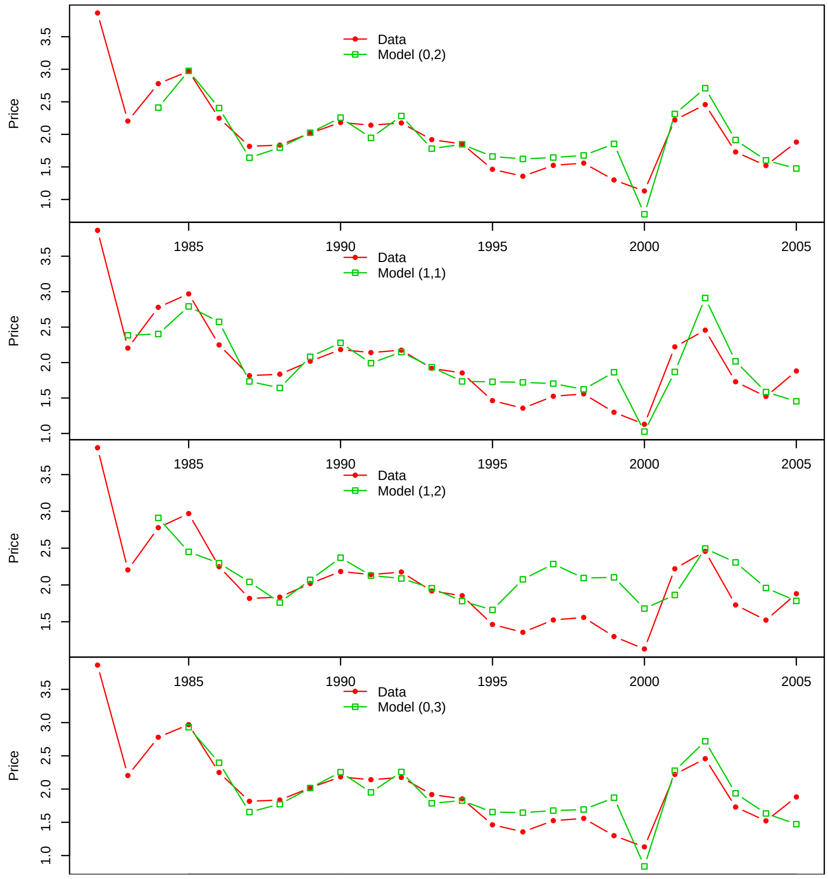


Table 18. Statistics for generalized least squared estimates

	Par	LogLik	Sigma	Lag 1	Lag 2	Lag 3	Lag 4
Model (0,0)	2	-17.5485	0.5027	0.5832	1.0201	0.7450	0.9012
Model (1,0)	3	-10.3114	0.3497	1.7281	2.8730	1.1315	0.9655
Model (0,1)	3	-7.4173	0.2838	1.1686	1.8289	1.1204	0.9438
Model (2,0)	4	-10.0513	0.3505	1.7435	2.7329	1.0537	0.9333
Model (1,1)	4	-5.1579	0.2617	1.8248	2.4718	1.2500	0.9732
Model (0,2)	4	-3.8654	0.2251	1.8856	1.8345	1.3978	1.3758
Model (1,2)	5	-5.9966	0.3949	0.8986	1.4164	1.0768	1.2005
Model (0,3)	5	-3.7112	0.2167	1.8340	1.7615	1.4962	1.3892

Par refers to number of parameters and "Lag *n*" relates to the Durbin-Watson statistic of the residual with lag *n*.

Model (0,0) show autocorrelation for lag 1 and lag 2. In improving this model with one more parameter the model (0,1), in compare with model (1,0), shows the highest likelihood and the smallest σ. However the model (0,1) still have autocorrelation and the model (1,0) have a negative autocorrelation for lag 2. Improvement of model (1,0) with one more autocorrelation term do not seem to yield a good result. When improving model (0,1) with one more parameter, model (0,2) shows a higher likelihood and lower σ than model (1,1), all Durbin-Watson statistics is better for model (0,2) too, therefore model (0,2) is preferred for the models with 4 parameters.

There seems to be no gain in adding one more parameter, the best model with 5 parameters is model (0,3), and here the likelihood is only slightly improved. Consequently model (0,2) is accepted as final model. Parameter estimates and test statistics for the model (0,2) is given in table 19. Both parameters is highly significant.

Table 19. Parameter estimates and statistics for the model (0,2)

Parameter	Estimate	Std.error	t-value	p-value
θ_1	1.9908			
θ_2	1.0000			
α	4.0104	0.2517	15.93	1.447e-13
β	-0.01309	0.001223	-10.70	3.473e-10

Multiple R-Squared: 0.7557 Adjusted R-squared: 0.7435
F-statistic: 61.8823 on 1 and 20 DF, p-value: 1.510e-07
Deviance: 34.7428 on 3 DF, p-value: 1.381e-07

The F and R statistics compare the residuals of the model with the residuals of a fix price model, $E(p) = \alpha$. As residuals is in model (0,2) used ν_t. As there in this model is no residual for the first two observations, the first two observations are left out in the estimation of the fixed price model. Note however; as there in the estimation of model (0,2), as object not is used minimum of the sums of squares, but maximum of likelihood, the general logic of variance analysis do not apply. However, the deviance statistics – minus 2 times the difference in loglikelihood – is asymptotic χ^2_{DF} distributed.

Conclusion

If the autocorrelation term is ignored the price can be predicted by

$$E(p_t) = \alpha + \beta h_t$$

with the parameters given in table 19. However the landing is referring to the landings in Denmark, not from the North Sea. Landings of herring in Denmark is in average 0.0574 of total catch in the North Sea (std.err. 0.02), it is therefore reasonable to anticipate only this fraction of the North Sea harvest will appear on the Danish marked and influence the price. The formula therefore has to be corrected:

$$\begin{aligned} E(p_t) &= \alpha + \beta h_t \\ &= \alpha + 0.0574\beta H_t \\ &= \alpha + B H_t \end{aligned}$$

where H is the total harvest in the North Sea and the B = –0.0007513 when H is measured in 1.000 ton.

Cost function Herring

Theory

Total cost for herring harvesting is expected to be of the form

$$C(H) = \alpha H^{\beta} \qquad (1)$$

Where α and β is parameters and H is total harvest of herring. The function is not expected to be a function of the biomass of the herring as the herring is shoaling. Other functional forms with a dependency on stock in different forms have been tested with out success. If the production is divided into to sectors the total cost can be written as

$$C = \alpha_i h_i^{\beta} + \alpha_j \left(H - h_i\right)^{\beta}$$

If the cost function is assumed equal for the two sectors i.e. $\alpha_i = \alpha_j$ we have

$$C = \alpha_i \left(h_i^{\beta} + \left(H - h_i\right)^{\beta}\right) \qquad (2)$$

Equitation (1) and (2) yields

$$\alpha_i = \frac{\alpha H^{\beta}}{h_i^{\beta} + \left(H - h_i\right)^{\beta}}$$

α and β can therefore be estimated from a single sector empirical cost:

$$\begin{aligned} C_i(h_i) &= \alpha_i h_i^{\beta} \\ &= \alpha \frac{h_i^{\beta} H^{\beta}}{\left(h_i^{\beta} + \left(H - h_i\right)^{\beta}\right)} \qquad (3) \end{aligned}$$

The accounting statistic for fishery in Denmark has as its basic unit a firm, normally consisting of one fishing vessel. The Danish fishing vessels catch a mixture of fish and operate in both the Baltic and the North See. The fishery in the North Sea is practiced by a lot of nations. The segment of the Danish fleet which operates partly in the North See and catch most of the herring is the segment called "Herring, mackerel and fish for reduction". This segment is in the statistics distinct form the vessels operating only with "Fish for reduction". Our approach is to use accounting data for the Danish "Herring, mackerel and fish for reduction" fleet and to estimate the total cost in the North Sea with the equation (3). Therefore the model is

$$E(C_t) = \alpha \frac{h_t^{\beta} H_t^{\beta}}{\left(h_t^{\beta} + \left(H_t - h_t\right)^{\beta}\right)}$$

Where $E(C_t)$ and h_t are the expected variable cost and harvest for the Danish "Herring, mackerel and fish for reduction" fleet in year t, and H_t is the total harvest of herring in the North Sea. The parameter α and β can then be used in equation (1) to extrapolate to total costs.

Data

The fishery account statistic from 1995-1998 (Statens Jordbrugs- og Fiskeriøkonomiske Institut, 1997*a,b*, 1998, 1999) have data for variable cost, gross output distributed according to species and an estimate of the fisherman's remuneration. In addition there is output figures for species in ton.

From 1999-2004 the account statistic (Statens Jordbrugs- og Fiskeriøkonomiske Institut, 2001; Fødevareøkonomisk Institut, 2005) have gross output grouped as "Herring and mackerel" and there is no figures for the physically output. To get the relevant figures for herring the fleets share of the total Danish fleets catch (data from: Fiskeridirektoratet, 2006*a*) is assumed fixed. The "Herring, mackerel and fish for reduction" fleets share of total Danish catch is estimated as the mean of the 1996-1998 data (mean share is 0.6398). To get the value of the herring catch the landing in ton is multiplied by the average price of herrings for that year (data from: Fiskeridirektoratet, 2006*a,b*)

As the "Herring, mackerel and fish for reduction" vessels is landing a variety of species, the variable cost for herring is calculated so the herrings share of cost equal herrings share of gross output. All data is in 1,000DKK and ton of landed herring for the segment in total. The variable cost in nominal prices is converted into real price with CPI (Danmarks Statistik, 2006) with 2004 as base, and converted into NOK by exchange rate 100DKK=90.9300NOK (1/6 2004). For the total harvest of herring in North Sea ICES Advisory Committee on Fishery Management (2005, Table 2.6.2.3 North Sea herring. STOCK SUMMARY) is used. The final data set is given in table 20.

Table 20.Landings in ton and variable cost in 1.000 NOK real price (2004) for the Danish herring fleet and the total harvest of herring in the North Sea in ton.

	Year	Landings	Variable cost	Harvest
1	1995	136290.00	199847.39	579371
2	1996	116237.80	135282.25	275098
3	1997	64237.80	83482.84	264313
4	1998	90421.00	127189.04	391628
5	1999	88028.99	110213.07	363163
6	2000	98471.95	70357.21	388157
7	2001	90543.59	99772.21	363343
8	2002	72035.35	73310.47	370941
9	2003	73458.37	79823.41	472587
10	2004	87536.95	69139.47	567252

Model

Table 21. Statistics from a nonlinear least square estimate of the model (equation 4)

	Estimate	Std. Error	t value	Pr(> | t |)
Alpha	0.02210115	0.06641652	0.3327658	0.7478635098
Beta	1.32953275	0.24968713	5.3247949	0.0007069434

Residual standard error: 23898 on 8 degrees of freedom
Multiple R-Squared: 0.6964 Adjusted R-squared: 0.6584
F-statistic: 18.3464 on 1 and 8 DF, p-value: 0.002676
Loglikelihood -113.89

The R and F statistics compare the residuals of the model with the residuals of the model $E(C) = \gamma$. Note however; as the later is not a sub model of the former the general logic of variance analysis do not apply.

A nonlinear least square estimate of the model (4) gives the results in table 21. Note that the t-test in the summary is a test with H_0: $\beta = 0$, where as the interesting hypothesis might be $\beta = 1$: this hypothesis can not be rejected. The α and β is highly (negative) correlated, therefore only one is significant. If β is exogenous the α is significant in an ordinary least square model with $\beta = 1.33$, see table 22.

Table 22. Statistics from an ordinary least square estimate with exogenous $\beta = 1.33$

	Estimate	Std Error	t value	Pr(> | t |)
Alpha	0.02197723	0.001427096	15.39996	8.974311e-08

Residual standard error: 22531 on 9 degrees of freedom
Multiple R-Squared: 0.6964 Adjusted R-squared: 0.6964
Loglikelihood -113.89

The R statistic compare the residuals of the model with the residuals of fixed cost model, $E(C)=\gamma$. Note however; as the later is not a submodel of the former the general logic of variance analysis do not apply. As the model have the same number of parameters as the fixed cost model there is no F statistics defined.

Notice that the t-test is for H_0: $\alpha = 0$, a more relevant test is to test if the cost is fixed, i.e. H_0: $E(C_t) = \gamma$ or if relative cost is fixed, i.e. H_0: $E(C_t) = \gamma h_t$. The number of parameters in the test models and in equation (4) with β as exogenous is the same (i.e. 1) so we can compare sigma and log likelihood with the models above, see table 23.

Table 23. The residual standard error and the log likelihood statistics from estimation of the following models Model A is non linear estimate of both α and β (2 parameters) and model B is liner model with exogenous β = 1.33 (1 parameter), both referring to the model (4). Models for fixed cost is $E(C_t)=\gamma$ and fixed relative cost is $E(C_t)=\gamma h_t$.

	sigma	loglik
Model A	23898	-113.89
Model B	22531	-113.89
Fixed cost	40889	-119.85
Fixed relative cost	24770	-114.84

Even though the difference between the fixed relative cost model and the model (4) not is big (and not significant in a quotient test), the proposed model is accepted as final model.

Conclusion

The expected variable cost in the North Sea herring fishery in 1,000 NOK real prices (2004) can be estimated by:

$$E(C) = \alpha H^{1.33}$$

where α = 0.021977 and H is total harvest of herring in ton. If harvest is measured in 1,000 ton and cost in million NOK the formula is the same just with $\alpha' = 1{,}000^{0.33}\alpha = 0.21477$.

Growth Function Cod and Herring

Data

Data for cod and for herring in North sea is used to estimate species interdependency (ICES Advisory Committee on Fishery Management, 2004, Table 3.4.9 Cod in Subarea IV and Divisions IIIa (Skagerrak) and VIId: Stock summary as estimated by ADAPT without discards; ICES Advisory Committee On Fishery Management, 2005, Table 2.6.2.3 North Sea herring. STOCK SUMMARY). Data is in 1.000 ton, growth at time t for species $i, j \in \{cod, herring\}$ is calculated as

$$g_{i,t} = X_{i,t+1} - X_{i,t} + h_{i,t}$$

Where X is biomass and h is harvest. The dataset is given in table 24.

Table 24. Biomass and growth for cod and herring in the North Sea

	Year	Cod biomass	Cod growth	Herring biomass	Herring growth
1	1960	NA	NA	3719.372	1314.803
2	1961	NA	NA	4337.975	739.378
3	1962	NA	NA	4380.653	855.792
4	1963	448.184	194.904	4608.645	888.691
5	1964	526.631	280.101	4781.336	419.745
6	1965	680.691	326.380	4329.881	147.157
7	1966	826.035	289.811	3308.238	403.083
8	1967	894.510	117.325	2815.821	400.110
9	1968	758.858	134.691	2520.431	102.463
10	1969	605.181	522.408	1905.094	563.507
11	1970	926.829	432.571	1921.901	490.625
12	1971	1133.276	-10.658	1849.426	220.143
13	1972	794.520	190.559	1549.469	103.996
14	1973	631.103	213.229	1155.965	239.922
15	1974	605.281	288.824	911.887	43.349
16	1975	679.826	109.775	680.136	-8.999
17	1976	584.356	444.801	358.337	26.608
18	1977	794.988	189.503	210.145	60.423
19	1978	775.337	290.855	224.568	168.164
20	1979	769.170	476.345	381.732	273.481
21	1980	975.542	138.436	630.113	598.986
22	1981	820.334	321.544	1158.335	859.395
23	1982	806.381	118.468	1842.851	1150.531
24	1983	621.598	329.604	2718.303	532.676
25	1984	691.915	19.863	2863.777	1025.805
26	1985	483.492	391.936	3460.951	623.627
27	1986	660.799	98.747	3470.798	1134.475
28	1987	555.493	72.530	3933.785	433.882
29	1988	411.811	178.747	3575.609	617.481
30	1989	406.318	57.372	3305.404	453.452
31	1990	323.754	104.047	2970.957	382.931
32	1991	302.487	242.958	2708.659	380.208
33	1992	442.967	85.072	2430.859	798.845
34	1993	414.019	301.812	2512.905	171.843
35	1994	594.082	105.932	2013.351	368.882
36	1995	589.380	33.831	1813.999	360.150
37	1996	487.115	212.138	1594.778	586.701
38	1997	572.933	-92.514	1906.381	357.721
39	1998	356.261	91.272	1999.789	679.636
40	1999	301.519	58.701	2287.797	939.325
41	2000	263.995	15.515	2863.959	759.383
42	2001	208.139	82.923	3235.185	1168.563
43	2002	241.430	-2.361	4040.405	186.258
44	2003	184.204	NA	3855.722	144.795
45	2004	NA	NA	3527.930	NA

Model

There is assumed a logistic growth function and an interdependency term of the form $\gamma X_{i,t} X_{j,t}$. The model is then

$$E(g_{i,t}) = \alpha_i X_{i,t} - \beta_i X_{i,t}^2 - \gamma_i X_{i,t} X_{j,t}$$

where $i, j \in \{cod, herring\}$. The two equations is fitted as a system with seemingly unrelated regression (`systemfit("SUR",)` in Hamann and Henningsen, 2006) and gives the following estimates and statistics given in table 25.

Table 25. Statistics for the model estimated using seemingly unrelated regression

Parameter	Estimate	Std.error	t-value	p-value
α_{cod}	0.7007	0.1702	4.116	0.0002067
β_{cod}	-0.0004745	0.0001842	-2.577	0.01410
γ_{cod}	-2.902e-05	3.086e-05	-0.9402	0.3532
$\alpha_{herring}$	0.4351	0.09118	4.772	2.848e-05
$\beta_{herring}$	-6.476e-05	1.940e-05	-3.339	0.001929
$\gamma_{herring}$	-7.379e-05	9.39e-05	-0.7857	0.437
Estimations statistics for cod: Residual standard error: 139.165768 on 37 degrees of freedom Multiple R-Squared: 0.140244 Adjusted R-Squared: 0.09377				
Estimations statistics for herring: Residual standard error: 298.950609 on 37 degrees of freedom Multiple R-Squared: 0.212386 Adjusted R-Squared: 0.169812				

The correlations of the residuals

	Cod	Herring
Cod	1.0000000	1.-0.0466624
Herring	-0.0466624	1.0000000

The R statistics compare the residuals of the models with the residuals of the model *E(g) = γ*. Note however; as the later is not a submodel of the former the general logic of variance analysis do not apply.

The interdependency term γ_{cod}= –2.902e-05 and $\gamma_{herring}$= –7.379e-05 are both negative suggesting that the species to some degree are competitors for the same resource. They are insignificant as well. As the idea of the model is to have an interdependence term, the model is accepted despite the insignificance.

Conclusion

The growth of herring and cod can be predicted as:

$$E(g_{cod}) = \alpha_{cod} X_{cod} - \beta_{cod} X_{cod}^{2} - \gamma_{cod} X_{cod} X_{herring}$$

$$E(g_{herring}) = \alpha_{herring} X_{herring} - \beta_{herring} X_{herring}^{2} - \gamma_{herring} X_{herring} X_{cod}$$

with the parameters given in the table 2.

If the prediction is written as

$$E(g_{cod}) = r_{cod} X_{cod}\left(1 - \frac{X_{cod}}{K_{cod}} - {}_{cod} X_{herring}\right)$$

$$E(g_{herring}) = r_{herring} X_{herring}\left(1 - \frac{X_{herring}}{K_{herring}} - {}_{herring} X_{cod}\right)$$

the parameters are as given in table 26.

Table 26. Parameters for the alternative formulation of the growth functions

Parameter	Estimate		
r_{cod}		0.7007	$year^{-1}$
K_{cod}		1477	$10^{3}ton$
κ_{cod}		4.142e-05	$10^{-3}ton^{-1}$
$r_{herring}$		0.4351	$year^{-1}$
$K_{herring}$		6719	$10^{3}ton$
$\kappa_{herring}$		0.0001696	$10^{-3}ton^{-1}$

References

Arnason, R., L. K. Sandal, S.I. Steinshamn, N. Vestergaard, S. Argansson and F. Jensen (2000). Comparative evaluation of the cod and herring fisheries in Denmark, Iceland and Norway. In: *TemaNord 2000:526*. Nordisk Ministerråd.

Arnason, Ragnar, Leif K. Sandal, Stein Ivar Steinshamn and Niels Vestergaard (2004). Optimal feedback controls: Comparative evaluation of the cod fisheries in denmark, iceland, and norway. *American Journal of Agricultural Economics*. Vol. 86, nr. 2, pp. 531–542

Danmarks Statistik (2006). *PRIS8: Forbrugerprisindeks, årsgennemsnit (1900=100)*. Danmarks Statistik. http://www.statistikbanken.dk

Fiskeridirektoratet (2000*). Fiskeristatistisk Årbog 2000 Yearbook of Fishery Satistics 2000*. Fiskeridirektoratet, København.

Fiskeridirektoratet (2001). *Fiskeristatistisk Årbog 2001 Yearbook of Fishery Statistics 2001*. Fiskeridirektoratet, København.

Fiskeridirektoratet (2002*). Fiskeristatistisk Årbog 2002 Yearbook of Fishery Statistics 2002*. Fiskeridirektoratet, København.

Fiskeridirektoratet (2003). *Fiskeristatistisk Årbog 2003 Yearbook of Fishery Statistics 2003*. Fiskeridirektoratet, København.

Fiskeridirektoratet (2004). *Fiskeristatistisk Årbog 2004 Yearbook of Fishery Statistics 2004*. Fiskeridirektoratet, København.

Fiskeridirektoratet (2006*a*). *Danske fiskeres fangster fra samtlige farvande fordelt på arter 1996-2005, Hel fisk i ton*. Fiskeridirektoratet. Quoted 20/6 2006. http://webfd.fd.dk/stat/Faste%20tabeller/Landinger-10aar/tab74b.html

Fiskeridirektoratet (2006*b*). *Danske fiskeres fangster fra samtlige farvande fordelt på arter 1996-2005, Værdi i 1.000 kr.*. Fiskeridirektoratet. Quoted 20/6 2006. http://webfd.fd.dk/stat/Faste%20tabeller/Landinger-10aar/tab74b.html

Fødevareøkonomisk Institut (2005*a*). Grupperet regnskabsstatistik: Økonomiske størrelsesklasser (1996-2004). Den Kgl. Veterinær- og Landbohøjskole.http://www.foi.kvl.dk/upload/foi/docs/statistik/fiskeri/regnskabsstatistik%20i%20regneark/grupperede%20tabeller/fart%C3%B8jsl%C3%A6ngde%20og%20-type.xls.

Fødevareøkonomisk Institut (2005*b*). Regnskabsstatistik på regneark, Fartøjslængde og -type (2000-2004). Den Kgl. Veterinær- og Landbohøjskole. http://www.foi.kvl.dk/upload/foi/docs/statistik/fiskeri/regnskabsstatistik%20i%20regneark/grupperede%20tabeller/fart%C3%B8jsl%C3%A6ngde%20og%20-type.xls.

Hamann, Jeff D. and Arne Henningsen (2006). *systemfit: Simultaneous Equation Estimation Package*. R package version 0.8-0. http://www.r-project.org, http://www.forestinformatics.com, http://www.arne-henningsen.de

ICES Advisory Committee on Fishery Management (2004). Report on the Assessment of Demersal Stocks in the North Sea and Skagerrak,7–16 September 2004, Bergen, Norway. In:

ICES CM 2005/ACFM:07. The International Council for the Exploration of the Sea, Copenhagen.

ICES Advisory Committee On Fishery Management (2005). ICES WGNSSK 2005, Report of the Herring Assessment Working Group for the Area South of 62ºN (HAWG), 6 – 15 September 2005, ICES Headquaters Copenhagen. In: *ICES CM 2005/ACFM:16*. The International Council for the Exploration of the Sea, Copenhagen.

Pinheiro, Jose, Douglas Bates, Saikat DebRoy, and Deepayan Sarkar (2006). *nlme: Linear and nonlinear mixed effects models*. R package version 3.1-79.

Statens Jordbrugs- og Fiskeriøkonomiske Institut (1997a). *Fiskeriregnskabsstatistik 1995, Account Statistics for Fishery 1995*. Ministeriet for Fødevarer, Landbrug og Fiskeri, København.

Statens Jordbrugs- og Fiskeriøkonomiske Institut (1997b). *Fiskeriregnskabsstatistik 1996, Account Statistics for Fishery 1996*. Ministeriet for Fødevarer, Landbrug og Fiskeri, København.

Statens Jordbrugs- og Fiskeriøkonomiske Institut (1998). *Fiskeriregnskabsstatistik 1997, Account Statistics for Fishery 1997*. Ministeriet for Fødevarer, Landbrug og Fiskeri, København. Statens Jord.

Statens Jordbrugs- og Fiskeriøkonomiske Institut (1999). *Fiskeriregnskabsstatistik 1998, Account Statistics for Fishery 1998*. Ministeriet for Fødevarer, Landbrug og Fiskeri, København.

Statens Jordbrugs- og Fiskeriøkonomiske Institut (2001). *Fiskeriregnskabsstatistik 1999, Account Statistics for Fishery 1999*. Ministeriet for Fødevarer, Landbrug og Fiskeri, København.

Appendix 4. The theoretical model

The objective is to discover the time path of harvest that maximises the following functional:

$$\int_0^{\infty} e^{-\delta t}\Pi(h,x)dt \qquad (1)$$

subject to $\dot{x} = f(x,h),\ x(0) = x_0,\ \lim_{t\to\infty} x(t) = x^* > 0.$

where x represents the fish stock biomass, h the flow of harvest, Π net revenues and $f(.,.)$ is a function representing biomass growth. Dots on tops of variables are used to denote time derivatives, and δ is the discount rate. x_0 represents the initial biomass and x^* some positive (equilibrium) biomass level to which the optimal program is supposed to converge.[2] The functions Π and f can in principle be any functions although it is henceforth assumed that they are sufficiently regular for both the problem and the results to be meaningful.

The current value Hamiltonian corresponding to problem may be written as:[3]

$$H = H(h,x,\lambda) = \Pi(h,x) + \lambda f(h,x),$$

where λ is the costate variable. Assuming an interior solution (i.e. positive biomass and harvest), the necessary or first-order conditions for solving the maximisation problem (Kamien and Schwartz, 1991) include:

$$H_h = 0,$$
$$\dot{\lambda} = \delta\,\lambda - H_x.$$

Upon differentiating the Hamiltonian function with respect to time, these conditions combined with the dynamic constraint in (1) yield[4]

$$\dot{H} = \delta \cdot \lambda \cdot \dot{x} \qquad (2)$$

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2 Indeed, the last constraint in (1), which can be derived as a transversality condition, may be regarded as the requirement of fishery sustainability.

3 It is assumed that the multiplier corresponding to the objective function, $\Pi(h,x)$, is unity.

4 $\dot{H} = H_h\dot{h} + H_x\dot{x} + H_\lambda\dot{\lambda}$. From the necessary conditions, $H_h = 0$, $H_x = \delta\lambda - \dot{\lambda}$. Finally, by the construction of the Hamiltonian function, $H_\lambda = \dot{x}$

The interior optimum condition, $H_h = 0$, implies that the costate variable, λ, can be rewritten as a function of x and h:

$$\lambda = -\frac{\Pi_h}{f_h} \equiv \Lambda(h,x).$$

As this is a known function (provided the functions Π and f are known), it can be used to eliminate the costate variable, λ, from the problem. More to the point, it is now possible to define the following new function different from the Hamiltonian but always equal to it in value:

$$P(h,x) = \Pi(h,x) + \Lambda(h,x) f(h,x). \qquad (3)$$

For fisheries management, and, indeed, the purposes of this paper, it is extremely useful to be able to express the optimal harvest at each point of time as a function of the fish stock biomass at that time. Let us refer to this as the function $h(x)$. In the optimal control literature, this is referred to as feedback control (Seierstad and Sydsæter 1987 p. 161, Kamien and Schwartz 1991 p. 262). So, we seek the feedback control, $h(x)$, for problem (1). Inserting this unknown function into (3) and differentiating with respect to time yields:

$$\dot{P} = \left(\frac{\partial P}{\partial x} + \frac{\partial P}{\partial h} \frac{\partial h}{\partial x} \right) \cdot \dot{x}.$$

But by construction $\dot{P} \equiv \dot{H}$. Hence, by (2) we obtain the first-order differential equation that can be used to determine the feedback control:

$$\frac{dP}{dx} \equiv \frac{\partial P}{\partial x} + \frac{\partial P}{\partial h} \frac{\partial h}{\partial x} = \delta \cdot \Lambda(h,x). \qquad (4)$$

Solving (4) or, if that is more convenient, (3) for the harvest, h, yields the desired feedback control. This, however, is not a trivial task in general.

In the special case where the rate of discount, $\delta = 0$, it is particularly easy to find the optimal feedback control. In this case $\frac{dP}{dx} = 0$ by (4). In other words, P is a constant. This corresponds to the well-known result that with zero discounting the maximised Hamiltonian is constant (Seierstad and Sydsæter, 1987, pp. 110-11). Obviously, if this constant can be determined, the feedback control is given implicitly by (3) and our problem is solved.[5] Now, the Hamiltonian can be interpreted as the rate of increase of total assets (Dorfman 1969). Profit maximisation requires us to make this as large as possible for as long as possible. The largest pos-

5 Of course, without discounting, the integral in (1) may not converge, but with the listed transversality condition in (1) this is not a problem. Although the integral may have an infinite value, there exists one control trajectory that maximizes the integral. This is the trajectory whose value in terms of the objective function ultimately catches up with the value from any other control trajectory (Seierstad and Sydsæter, 1987, pp. 231-3).

sible sustainable value of the Hamiltonian is given by the maximum of the sustainable net revenue defined as

$$S(x) = \Pi(h, x)\big|_{\dot{x}=0} \quad (5)$$

which is a function of *x* only as *f(h,x)* = 0 can be used to eliminate *h*. Note that *S* is simply the net revenue that can be obtained by fixing the stock at any level. When $\delta = 0$, there is no discounting of the future and obviously the constant we are seeking is $P = P_0 = \max[S(x)]$. This constant substituted for the left-hand side of (3) gives the optimal feedback control as an ordinary algebraic equation (not a differential equation). This equation can subsequently be used for comparative dynamics and sensitivity analysis. Note, however, that the feedback control itself, *h*(*x*), has normally to be found by numerical means, although in certain special cases it is possible to obtain explicit solutions.

In the more general case, where $\delta > 0$, it is unavoidable to seek the solution on the basis of the differential equation given in (4). This equation can either be solved numerically for the optimal feedback control or perturbation methods can be used in order to find closed form solutions if that is required, see, e.g., Sandal and Steinshamn (1997a).

Stochastic model
As mathematical modelling framework we choose stochastic optimal control theory with aggregated stochastic differential equations (SDE) in continuous time and state. The SDEs represent the "bio-political" regeneration process of our perception of the marine resources under consideration.

The aggregated biomass is described by SDEs of the form

$$dx_t = [f(t, x_t) - h_t]dt + \sigma(t, x_t)dB_t . \quad (1)$$

x_t is a representative measure of a stock (e.g. total biomass), $f(\bullet)$ is the natural regeneration function or the average incremental surplus growth of the stock with zero fishing effort. The volatility $\sigma(\bullet)$ of the process is almost surely dependent on the level of the resource and represents the aggregation of the intrinsic biological stochasticity combined with structural uncertainty in the model due to our lack of knowledge as well as level of aggregation. The quantities dt and dB_t are incremental time steps and the basic incremental Brownian motion with variation dt .

The strength of this approach is that it produces an adaptive harvest policy directly dependent on the underlying functions describing the natural surplus growth as well as the volatility. Thereby we can make reasonable statements about structural stability and perform sensitivity analysis of the suggested policies.

The bio-political objective is to maximize some expected discounted utility stream generated from the harvesting of the marine resources. This stochastic optimisation problem may need non-economic restrictions in order to ensure that fishing effort is not too high on small stocks that are not economically protected by their intrinsic costs profiles (such as bottom trawl fisheries).

Typically for economically protected species we get an objective of the form

$$\sup_{h_t \in P} E\left\{ \int_0^\infty \Pi\,(t, x_t, h_t)\,dt \right\} \tag{2}$$

That is, we maximize the expected value of an infinite horizon utility stream with density $\Pi(\bullet)$, by choosing a harvest rate h_t from the space of admissible policies *P*. The solution is constructed through the Hamilton-Jacobi-Bellman (HJB) equation for the optimal value function $V(s, y)$, defined as the value of (2) for a process where $x_s = y$ at a particular time $t = s$. The nature of the problem may be of some irregularity. We may then apply the modern notion of solution known as "viscosity solution". This is a particular form of weak solutions to HJB partial differential solution[6] given by

$$\frac{\partial V}{\partial s} + \sup_{h \in P}\left\{ \Pi(s, y, h) + \frac{\partial V}{\partial y} \cdot (f(s, y) - h) + \frac{1}{2}\sigma^2(s, y, h)\frac{\partial^2 V}{\partial y^2} \right\} = 0 \tag{3}$$

Appropriate boundary conditions and restrictions must be imposed.

The current value of the utility function, $\Pi(\bullet)$, is usually represented by the net cash flow derived from the fishery. It is typically a non-linear function of the harvest policy. This ensures that the optimal policy is not analogous to a bang-bang policy.

6 An advanced textbook introducing this modern solution concept is e.g. Bardi and Capuzzo-Dolcetta (1997).

References

Dorfman, R. 1969. An Economic Interpretation of Optimal Control Theory, *American Economic. Review* 59, pp. 817-831

Kamien, M.I., and Schwartz, N.L., 1991, *Dynamic optimisation: the calculus of variations and optimal control in economics and management* , Amsterdam, North-Holland.

Sandal, L. K. and S. I. Steinshamn, 1997a. A Feedback Model for the Optimal Management of Renewable Natural Capital Stocks. *Canadian Journal of Fisheries and Aquatic Sciences* 54: 2475-82.

Seieerstad, A. and K. Sydsæter. 1987. *Optimal Control Theory with Economic Applications*. North Holland, New York.